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On the stability of the solution mapping for parametric traffic network problems

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Abstract

In this paper, we introduce the parametric traffic network problems. Afterward, a key hypothesis is introduced by virtue of a parametric gap function to considered problems, and we prove that this hypothesis is not only sufficient but also necessary for the Hausdorff lower semicontinuity and Hausdorff continuity of the solution mapping for parametric traffic network problems.

Keywords: Traffic network problems; Hausdorff lower semicontinuity; Continuity; Hausdorff continuity

1. Introduction

In 1997, Zhao [17] studied the parametric optimization problem and proved that the assumption (H_1) is a sufficient and necessary condition for the Hausdorff lower semicontinuity of this problem. In 2005, Kien [12] also studied the same problem as Zhao [17], and proved the main results of Zhao [17] under weaker assumptions. Then, Li and Chen [13] and Chen et al. [5] introduced a condition (H_g) which is similar to the one given in [12,17] and a sufficient condition for the Hausdorff lower semicontinuity of the solution set for vector variational inequalities also investigated. Recently, Zhong and Huang [18] introduced the parametric set-valued weak

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vector variational inequalities in Banach spaces and proved that condition (H'_g) is sufficient and necessary for the Hausdorff lower semicontinuity of the solutions for these problems.

On the other hand, the network equilibrium model was introduced and investigated by Wardrop [16] for transportation network problems. In the intensive development of traffic network problems, Smith [15] made a turning point by proving that the Wardrop equilibria flows of the network are the solutions of the variational inequality corresponding to the network problem. Since then, there has been an increasing interest by many authors in different topics for these problems such as the existence conditions (see e.g. [6,7,9,10]), the well-posedness (see e.g. [1,11]), and stability of solutions (see e.g. [2,3]). However, to the best of our knowledge, up to now there have not been any papers devoted to the stability properties in the sense of Berge and Hausdorff for the solution mapping to the traffic network problems for strong equilibrium flow vectors by gap function method.

Motivated and inspired by the work mentioned above, in this paper, we introduce the parametric traffic network problems. Then, we study the stability properties such as the Hausdorff lower semicontinuity, continuity and Hausdorff continuity of the solution mapping for this problem by gap function method. The results presented in this paper are new and different from some main results in the literature.

Now, we pass to our problem setting, which was discussed by many authors (see e.g. [2,3,8–10,14,16]) and the references therein. Consider a transportation network $L = (N, A)$, where N denotes the set of nodes and A denotes the set of arcs. Let $Q = (Q_1, Q_2, \ldots, Q_n)$ be the set of origin–destination pairs (O/D pairs in short). Assume that the pair $Q_i, i = 1, 2, \ldots, n$ is connected by a set S_i of paths and S_i contains $s_i \geq 1$ paths. Let $F = (F_1, F_2, \ldots, F_m)$ be the paths vector flow, where $m = \sum_{i=1}^{n} s_i$. Let the capacity restriction be

$$F \in C := \{F \in \mathbb{R}^m : 0 \leq \omega_p \leq F_p \leq \Omega_p, p = 1, 2, \ldots, m\},$$

where ω_p and Ω_p are given real numbers, $C \subseteq \mathbb{R}^m$ be a nonempty set. Assume further that the travel cost on the path flow F_p, $p = 1, 2, \ldots, m$, depends on the whole path vector flow F and $T_p(F, \gamma) \in \mathbb{R}_+$, where $\gamma \in \Gamma$ is a perturbing parametric. Then we have a multifunction $T : \mathbb{R}^m \times \Gamma \rightrightarrows \mathbb{R}^m_+$ with $T(F, \gamma) = (T_1(F, \gamma), T_2(F, \gamma), \ldots, T_m(F, \gamma))$.

The following is extension of the Wardrop equilibrium to the case of multivalued costs.

A path flow vector $\bar{F}$ is said to be a weak equilibrium flow vector if

$$\forall Q_i, \forall \xi, \tau \in S_i, \exists z \in T(\bar{F}, \gamma) \text{ such that } [z_\xi < z_\tau] \Rightarrow [\bar{F}_\xi = \Omega_\xi \text{ or } \bar{F}_\tau = \omega_\tau].$$

A path flow vector $\bar{\Psi}$ is said to be a strong equilibrium flow vector if

$$\forall Q_i, \forall \xi, \tau \in S_i, \forall z \in T(\bar{F}, \gamma) \text{ such that } [z_\xi < z_\tau] \Rightarrow [\bar{F}_\xi = \Omega_\xi \text{ or } \bar{F}_\tau = \omega_\tau].$$

Suppose the travel demand ψ_i of the O/D pair $Q_i, i = 1, 2, \ldots, n$, depend on the weak (or strong) equilibrium flows $\bar{F}$. Hence, considering all the O/D pairs, we have a mapping $\psi : \mathbb{R}^m_+ \to \mathbb{R}^n_+$. We use the Kronecker notation

$$\phi_{i\tau} = \begin{cases} 1 & \text{if } \tau \in S_i, \\ 0 & \text{if } \tau \notin S_i. \end{cases}$$

and

$$\phi = \{\phi_{i\tau}\}, \quad i = 1, 2, \ldots, n, \text{ and } \tau = 1, 2, \ldots, m.$$

Then, the set of the path flows satisfying exactly the demands is

$$\{F \in \mathbb{R}^m | F \in C, \phi F = \psi(\bar{F}, \gamma)\}.$$

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