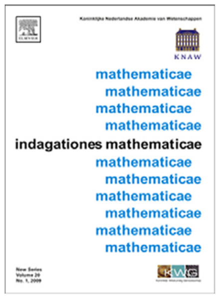
Accepted Manuscript



Factors of generalised polynomials and automatic sequences

Jakub Byszewski, Jakub Konieczny

PII: S0019-3577(18)30084-3
DOI: https://doi.org/10.1016/j.indag.2018.03.003
Reference: INDAG 561

To appear in: *Indagationes Mathematicae*

Received date : 9 February 2018
Accepted date : 16 March 2018

Please cite this article as: J. Byszewski, J. Konieczny, Factors of generalised polynomials and automatic sequences, *Indagationes Mathematicae* (2018), https://doi.org/10.1016/j.indag.2018.03.003

FACTORS OF GENERALISED POLYNOMIALS AND AUTOMATIC SEQUENCES

JAKUB BYSZEWSKI[2] AND JAKUB KONIECZNY[1,2]

Abstract. The aim of this short note is to generalise the result of Rampersad–Shallit saying that an automatic sequence and a Sturmian sequence cannot have arbitrarily long common factors. We show that the same result holds if a Sturmian sequence is replaced by an arbitrary sequence whose terms are given by a generalised polynomial (i.e., an expression involving algebraic operations and the floor function) that is not periodic except for a set of density zero.

A Sturmian sequence is defined as an infinite word with values 0 and 1 that encodes the set of times at which the orbit of a point with respect to an irrational rotation by θ hits a given arc of length θ. It is well-known that a Sturmian sequence is not automatic, i.e., it cannot be produced by a finite automaton that reads the base-k digits of the input in some fixed base $k \geqslant 2$. In a recent note [RS18], Rampersad and Shallit have shown that not only is it impossible for automatic and Sturmian words to coincide—their common factors (i.e., finite blocks of consecutive symbols) have in fact little in common.

Theorem 1 (Rampersad–Shallit)**.** *Let x be a k-automatic sequence and let a be a Sturmian sequence. There exists a constant C (depending on x and a) such that if x and a have a factor in common of length n then $n \leqslant C$.*

A Sturmian sequence can be equivalently defined as an infinite word $a_0 a_1 a_2 \cdots$ of the form

$$a_n = \lfloor \alpha(n+1) + \rho \rfloor - \lfloor \alpha n + \rho \rfloor - \lfloor \alpha \rfloor,$$

where $\alpha, \rho \in \mathbb{R}$ with $\alpha \notin \mathbb{Q}$. An expression of this form is a very simple example of a *generalised polynomial*, i.e., a function $a \colon \mathbb{N}_0 \to \mathbb{R}$ given by an expression involving real constants, the algebraic operations of addition and multiplication along with the (possibly iterated) use of the floor function. By a result of Bergelson–Leibman [BL07], generalised polynomials are intimately related to dynamics on nilmanifolds. (In the case of a Sturmian sequence the corresponding nilmanifold is the circle with the irrational rotation by θ.) In a recent work, we have shown that generalised polynomials cannot be automatic unless they are periodic outside of a set of density zero (for this and related results see [BK17] and [BK16]). It is therefore natural

2010 *Mathematics Subject Classification.* Primary: 11B85, 37A45.
Key words and phrases. Generalised polynomials, automatic sequences, nilmanifolds.