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Factors of generalised polynomials and automatic sequences

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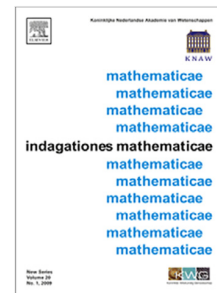
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11 **FACTORS OF GENERALISED POLYNOMIALS AND**  
12 **AUTOMATIC SEQUENCES**  
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20 **ABSTRACT.** The aim of this short note is to generalise the result of  
21 Rampersad–Shallit saying that an automatic sequence and a Sturmian  
22 sequence cannot have arbitrarily long common factors. We show that  
23 the same result holds if a Sturmian sequence is replaced by an arbitrary  
24 sequence whose terms are given by a generalised polynomial (i.e., an  
25 expression involving algebraic operations and the floor function) that is  
26 not periodic except for a set of density zero.

27 A Sturmian sequence is defined as an infinite word with values 0 and 1  
28 that encodes the set of times at which the orbit of a point with respect to  
29 an irrational rotation by  $\theta$  hits a given arc of length  $\theta$ . It is well-known  
30 that a Sturmian sequence is not automatic, i.e., it cannot be produced by a  
31 finite automaton that reads the base- $k$  digits of the input in some fixed base  
32  $k \geq 2$ . In a recent note [RS18], Rampersad and Shallit have shown that not  
33 only is it impossible for automatic and Sturmian words to coincide—their  
34 common factors (i.e., finite blocks of consecutive symbols) have in fact little  
35 in common.  
36

37 **Theorem 1** (Rampersad–Shallit). *Let  $x$  be a  $k$ -automatic sequence and let*  
38  *$a$  be a Sturmian sequence. There exists a constant  $C$  (depending on  $x$  and*  
39  *$a$ ) such that if  $x$  and  $a$  have a factor in common of length  $n$  then  $n \leq C$ .*  
40

41 A Sturmian sequence can be equivalently defined as an infinite word  
42  $a_0a_1a_2 \cdots$  of the form  
43

$$44 \quad a_n = \lfloor \alpha(n+1) + \rho \rfloor - \lfloor \alpha n + \rho \rfloor - \lfloor \alpha \rfloor,$$

45 where  $\alpha, \rho \in \mathbb{R}$  with  $\alpha \notin \mathbb{Q}$ . An expression of this form is a very simple  
46 example of a *generalised polynomial*, i.e., a function  $a: \mathbb{N}_0 \rightarrow \mathbb{R}$  given by  
47 an expression involving real constants, the algebraic operations of addition  
48 and multiplication along with the (possibly iterated) use of the floor func-  
49 tion. By a result of Bergelson–Leibman [BL07], generalised polynomials are  
50 intimately related to dynamics on nilmanifolds. (In the case of a Sturmian  
51 sequence the corresponding nilmanifold is the circle with the irrational ro-  
52 tation by  $\theta$ .) In a recent work, we have shown that generalised polynomials  
53 cannot be automatic unless they are periodic outside of a set of density zero  
54 (for this and related results see [BK17] and [BK16]). It is therefore natural  
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58 *2010 Mathematics Subject Classification.* Primary: 11B85, 37A45.

59 *Key words and phrases.* Generalised polynomials, automatic sequences, nilmanifolds.  
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