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Hypercyclic composition operators on the little Bloch space and the Besov spaces

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Abstract

Let $S(\mathbb{D})$ be the collection of all holomorphic self-maps on $\mathbb{D}$ of the complex plane $\mathbb{C}$, and C_φ the composition operator induced by $\varphi \in S(\mathbb{D})$. We obtain that there are no hypercyclic composition operators on the little Bloch space $\mathcal{B}_0$ and the Besov space B_p.

Keywords: Hypercyclic; Composition operator; Bloch space; Besov space

1. Introduction

Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ be the unit disk in the complex plane $\mathbb{C}$ and $S(\mathbb{D})$ be the collection of all holomorphic self-maps on $\mathbb{D}$. We denote $dA(z) = dxdy$ the Lebesgue area measure on $\mathbb{C}$. For the composition operator C_φ induced by $\varphi \in S(\mathbb{D})$ is defined as

$$C_\varphi f(z) = f \circ \varphi(z), \quad f \in H(\mathbb{D}), \quad z \in \mathbb{D}.$$

The one-to-one holomorphic functions that map $\mathbb{D}$ onto itself, called the *Möbius* transformations, and denoted by $\mathcal{M}$ (also $Aut(\mathbb{D})$), have the form $\lambda\varphi_a$, where $|\lambda| = 1$ and φ_a is the basic

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conformal automorphism defined by

$$\varphi_a(z) = \frac{a-z}{1-\bar{a}z}, \quad z \in \mathbb{D},$$

for $a \in \mathbb{D}$. The following identities are easily verified:

$$1 - |\varphi_a(z)|^2 = \frac{(1-|a|^2)(1-|z|^2)}{|1-\bar{a}z|^2}$$

and

$$(1-|z|^2)|\varphi_a'(z)| = 1 - |\varphi_a(z)|^2. \tag{1.1}$$

A linear space X of analytic functions on the open unit disk $\mathbb{D}$ is said to be *Möbius-invariant*, if $f \circ S \in X$ for all $f \in X$ and all $S \in \mathcal{M}$ and X has a seminorm $\| \ \|_X$ such that $\|f \circ S\|_X = \|f\|_X$ for each $f \in X$ and each $S \in \mathcal{M}$.

The well-known *Möbius-invariant* function space — the Besov spaces B_p $(1 < p < \infty)$ are defined as follows

$$B_p = \{f \in H(\mathbb{D}) : \ \|f\|_{B_p}^p = \int_{\mathbb{D}} |f'(z)|^p (1-|z|^2)^{p-2} dA(z) < \infty\},$$

that is, $f \in B_p$ if and only if the function $(1-|z|^2)f' \in L^p(\mathbb{D}, d\lambda)$, where

$$d\lambda(z) = \frac{dA(z)}{(1-|z|^2)^2}.$$

Although the measure λ is not a finite measure on $\mathbb{D}$, it is a *Möbius-invariant*. Indeed, by (1.1)

$$d\lambda(\varphi_a(z)) = \frac{|\varphi_a'(z)|^2}{(1-|\varphi_a(z)|^2)^2} dA(z) = \frac{dA(z)}{(1-|z|^2)^2} = d\lambda(z).$$

Hence we have the following change-of-variable formula

$$\int_{\mathbb{D}} f \circ \varphi_a(z) d\lambda(z) = \int_{\mathbb{D}} f(u) d\lambda(u),$$

for every positive measurable function f on $\mathbb{D}$, from which it is easily seen that

$$\|f \circ \varphi_a\|_{B_p} = \|f\|_{B_p}, \tag{1.2}$$

and the above identity also holds for $S = \lambda\varphi_a \in \mathcal{M}$ with $|\lambda| = 1$. Thus

$$\text{if } f \in B_p \ \text{ then } \ f \circ S \in B_p \ \text{ for all } \ S \in \mathcal{M}.$$

That is, B_p $(1 < p < \infty)$ are *Möbius-invariant* spaces.

For $p = 1$, the Besov space B_1 consists of the analytic functions f on $\mathbb{D}$ that admit the representation

$$f(z) = \sum_{n=1}^{\infty} a_n \varphi_{\lambda_n}(z), \quad z \in \mathbb{D},$$

where $\{a_n\} \in l^1$ and $\lambda_n \in \mathbb{D}$ for $n \in \mathbb{N}$. The norm in B_1 is defined as

$$\|f\|_{B_1} = \inf \left\{ \sum_{n=1}^{\infty} |a_n| : \ f(z) = \sum_{n=1}^{\infty} a_n \varphi_{\lambda_n}(z), z \in \mathbb{D} \right\}.$$

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