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Fite–Hille–Wintner-type oscillation criteria for second-order half-linear dynamic equations with deviating arguments

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Abstract

We study oscillatory behavior of solutions to a class of second-order half-linear dynamic equations with deviating arguments under the assumptions that allow applications to dynamic equations with delayed and advanced arguments. Several improved Fite–Hille–Wintner-type criteria are obtained that do not need some restrictive assumptions required in related results. Illustrative examples and conclusions are presented to show that these criteria are sharp for differential equations and provide sharper estimates for oscillation of corresponding q-difference equations.

Keywords: Advanced argument; Delayed argument; Half-linear; Oscillation behavior; Second-order dynamic equation; Time scale

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1. Introduction

In this paper, we are concerned with the oscillatory behavior of a class of second-order half-linear functional dynamic equations

$$\left[r(t)\phi_\alpha\left(x^\Delta(t)\right)\right]^\Delta + q(t)\phi_\alpha\left(x(g(t))\right) = 0 \tag{1.1}$$

on an arbitrary time scale $\mathbb{T}$ unbounded above, where $t \in [t_0,\infty)_\mathbb{T} := [t_0,\infty) \cap \mathbb{T}$, $t_0 \geq 0$, $t_0 \in \mathbb{T}$, $\phi_\alpha(u) := |u|^{\alpha-1}u$, $\alpha > 0$, r is a positive rd-continuous function on $\mathbb{T}$ such that $r^\Delta \geq 0$, q is a positive rd-continuous function on $\mathbb{T}$, and $g : \mathbb{T} \to \mathbb{T}$ is a rd-continuous function satisfying $\lim_{t\to\infty} g(t) = \infty$. Analysis of qualitative properties of (1.1) is important not only for the sake of further development of the oscillation theory, but for practical reasons too. In fact, the study of half-linear equations has become an important area of research due to the fact that such equations arise in a variety of real world problems such as in the study of p-Laplace equations, non-Newtonian fluid theory, and the turbulent flow of a polytrophic gas in a porous medium; see Agarwal et al. [2,4].

For an excellent introduction to the calculus on time scales; see Bohner and Peterson [8] and Hilger [18]. We assume that the reader is familiar with the basic facts of time scales and time scale notation. By a solution of (1.1) we mean a nontrivial real-valued function $x \in \mathrm{C}^1_{\mathrm{rd}}[T_x,\infty)_\mathbb{T}$, $T_x \in [t_0,\infty)_\mathbb{T}$ such that $r\phi_\alpha(x^\Delta) \in \mathrm{C}^1_{\mathrm{rd}}[T_x,\infty)_\mathbb{T}$ and x satisfies (1.1) on $[T_x,\infty)_\mathbb{T}$, where C_{rd} is the set of right-dense continuous functions. A solution x of (1.1) is termed oscillatory if it is neither eventually positive nor eventually negative; otherwise, we call it nonoscillatory. The solutions vanishing in some neighborhood of infinity will be excluded from our consideration.

In what follows, we state some oscillation results for differential equations that will be related to our oscillation results for (1.1) on time scales and explain the important contributions of this paper. Fite [14] studied the oscillatory behavior of solutions to the second-order linear differential equation

$$x''(t) + q(t)x(t) = 0, \quad q(t) > 0, \tag{1.2}$$

and showed that if

$$\int_{t_0}^\infty q(s)\mathrm{d}s = \infty, \tag{1.3}$$

then every solution of equation (1.2) is oscillatory. Hille [19] improved condition (1.3) and proved that if

$$\liminf_{t\to\infty} t\int_t^\infty q(s)\mathrm{d}s > \frac{1}{4}, \tag{1.4}$$

then all solutions of (1.2) are oscillatory. Erbe [9] generalized the Hille-type condition (1.4) to the delay equation

$$x''(t) + q(t)x(g(t)) = 0, \quad q(t) > 0, \quad g(t) \leq t, \tag{1.5}$$

and obtained that if

$$\liminf_{t\to\infty} t\int_t^\infty \frac{g(s)}{s}q(s)\mathrm{d}s > \frac{1}{4}, \tag{1.6}$$

then every solution of (1.5) is oscillatory. Ohriska [21] proved that, if

$$\limsup_{t\to\infty} t\int_t^\infty \frac{g(s)}{s}q(s)\mathrm{d}s > 1, \tag{1.7}$$

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