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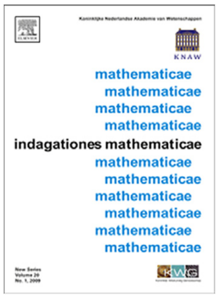

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Stability results for nonlinear functional differential equations using fixed point methods[1]

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Abstract

We present new conditions for stability of the zero solution for three distinct classes of scalar nonlinear delay differential equations. Our approach is based on fixed point methods and has the advantage that our conditions neither require boundedness of delays nor fixed sign conditions on the coefficient functions. Our work extends and improves a number of recent stability results for nonlinear functional differential equations in a unified framework. A number of examples are given to illustrate our main results.

Keywords: Fixed point theory, asymptotic stability, contraction mapping principle, (neutral) integro-differential equation, variable delay.

1. Introduction and main results

Lyapunov's direct method provides simple geometric theorems for deciding the stability or instability of an equilibrium point of a differential equation. However, in the context of functional differential equations, Lyapunov's direct method is not always as effective, in particular if the delay is unbounded or if the differential equation has unbounded terms. Therefore, it was recently proposed by Burton [6] and co-workers to use fixed point methods as an alternative. While Lyapunov's direct method usually requires pointwise conditions, fixed point methods need conditions of an averaging nature, and, therefore, can handle various delays or unbounded terms more easily.

A typical stability result based on fixed point theory arguments follows a number of standard arguments adapted to the special structure of the equation under consideration. This leads to many different results in the literature for different classes of equations, for example, with time dependent delays, distributed delays, neutral terms, and certain nonlinearities, see [2-14]. The aim of this paper is to study the approach using fixed point theory in a systematic way and to unify recent results in the literature by considering three general classes of equations. For each of these classes of equations, we combine different techniques to prove new stability theorems. In addition, we present a number of examples to illustrate our results.

The first class of functional differential equations that we will study in this paper is of the form

$$x'(t) = -\int_{t-r(t)}^{t} a(t,s)g(s,x(s))\,ds, \tag{1}$$

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