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Intuitionism and effective descriptive set theory

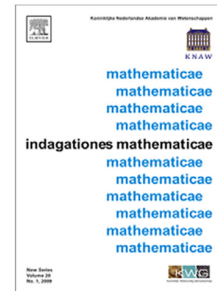
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INTUITIONISM AND EFFECTIVE DESCRIPTIVE SET THEORY

JOAN R. MOSCHOVAKIS AND YIANNIS N. MOSCHOVAKIS

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Our very eloquent charge from Jan van Mill was to “draw a line to Brouwer” from descriptive set theory, but this proved elusive: in fact there are few references to Brouwer, in Lusin [1928] and Lusin [1930], none of them substantial; and even though Brouwer refers to Borel, Lebesgue and Hadamard in his early papers, it does not appear that he was influenced by their work in any substantive way.¹ We have not found any references by him to more developed work in descriptive set theory, after the critical Lebesgue [1905]. So instead of looking for historical connections

¹Cf. Michel [2008], who traces the origin and dismisses the significance of terms like “semi-” or “pre-” intuitionists.

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