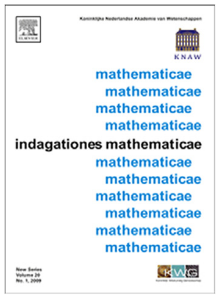
Accepted Manuscript



Operator norm and lower bound of four-dimensional matrices

Gholamreza Talebi

PII: S0019-3577(17)30085-X
DOI: http://dx.doi.org/10.1016/j.indag.2017.08.004
Reference: INDAG 504

To appear in: *Indagationes Mathematicae*

Received date : 13 April 2017
Revised date : 12 July 2017
Accepted date : 22 August 2017

Please cite this article as: G. Talebi, Operator norm and lower bound of four-dimensional matrices, *Indagationes Mathematicae* (2017), http://dx.doi.org/10.1016/j.indag.2017.08.004

OPERATOR NORM AND LOWER BOUND OF FOUR-DIMENSIONAL MATRICES

GHOLAMREZA TALEBI

ABSTRACT. Motivated by the work "Hardy's inequality in several variables" in *J. Math. Anal. Appl.*, 252 (2) (2000), 989-993, an extension of Copson's discrete inequality to multiple series is obtained. As an application of such extension, we consider the boundedness problem of four-dimensional matrices on the space $\mathcal{L}_p$ of double sequences. The operator norms of Cesàro, Copson and Nörlund four-dimensional matrices as operators selfmap of $\mathcal{L}_p$ are established. Further, a general upper estimate and lower estimate is obtained for the lower bounds of some four-dimensional matrices.

1. Introduction and Preliminaries

By Ω, we denote the space of all real or complex valued double sequences which is the vector space with coordinatewise addition and scalar multiplication. Any vector subspace of Ω is called as the double sequence space. Consider a sequence $x = (x_{n,m}) \in \Omega$. If there is an $\ell \in \mathbb{C}$ such that for each $\varepsilon > 0$ there exists $n_0 = n_0(\varepsilon) \in \mathbb{N}$ such that

$$|x_{n,m} - \ell| < \varepsilon$$

for all $n, m > n_0$, then we call that the double sequence $x = (x_{n,m})$ is convergent in the Pringsheim's sense to the limit ℓ and write $p-\lim_{n,m\to\infty} x_{n,m} = \ell$, [5], where $\mathbb{C}$ denotes the complex field. Also, we say that

$$s = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} x_{n,m},$$

if the sequence of partial sums $(s_{n,m})$, defined by

$$s_{n,m} := \sum_{j=0}^{n}\sum_{k=0}^{m} x_{j,k}, \quad (n, m = 0, 1, 2, ...),$$

is convergent in the Pringsheim's sense to limit s. In such a case, we say that the double series converges to s. A double sequence $x = (x_{nm})$ of real or complex numbers is said to be bounded if

$$\|x\|_\infty := \sup_{n,m} |x_{nm}| < \infty.$$

The space of bounded double sequences is denoted by $\mathcal{M}_u$. Also the space $\mathcal{L}_p$ of double sequences [1] is defined by

$$\mathcal{L}_p = \left\{ (x_{nm}) \in \Omega, \sum_{n,m} |x_{nm}|^p < \infty \right\}, \quad (1 \le p < \infty).$$

2000 *Mathematics Subject Classification.* Primary: 46A20, Secondary:26D15 .
Key words and phrases. Operator norm; four-dimensional matrix.