INDAG: 513 Model 1 pp. 1–18 (col. fig: NIL)

ARTICLE IN PRESS

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ScienceDirect

Indagationes Mathematicae xx (xxxx) xxx–xxx

indagationes mathematicae

www.elsevier.com/locate/indag

On the behavior of resonant frequencies in the presence of small anisotropic imperfections

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Received 22 February 2016; received in revised form 22 July 2017; accepted 12 September 2017

Communicated by R.C.A.M. van der Vorst

Abstract

In this paper we provide a rigorous derivation of an asymptotic formulae for perturbations in the resonant frequencies of the two-(or three-) dimensional Laplacian operator under geometric variation of the domain. The asymptotic expansion is developed in the presence and with respect to size of the (anisotropic) imperfections of small shapes having constitutive parameters different from the background conductivity. The main feature of the method is to yield a robust procedure making it possible to recover information about the location, shape, and material properties of the anisotropic imperfections.

Keywords: Resonant frequencies; Small perturbations; Anisotropic imperfections; Integral operators; Asymptotic expansion

1. Introduction

In this paper we discuss resonance problem associated with the Laplacian operator in a domain deformed by the presence of small (anisotropic) imperfections and when the scale factor goes to zero. The geometry of each imperfection takes the form ϵB where B is some bounded smooth domain. The goal then is to find an asymptotic expansion for the resonance values of such domain, with the intention of using this expansion as an aid in identifying the

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http://dx.doi.org/10.1016/j.indag.2017.09.007

Please cite this article in press as: M. Gozzi, A. Khelifi, On the behavior of resonant frequencies in the presence of small anisotropic imperfections, Indagationes Mathematicae (2017), http://dx.doi.org/10.1016/j.indag.2017.09.007.

imperfections. That is, we would like to find a method for determining the locations and/or shape of small (anisotropic) imperfections by taking resonance measurements. For the case of isotropic imperfections, one can refer to [1,2,4,13].

Theoretical modeling of the interaction between harmonic waves and anisotropic objects is of interest for many physical applications, such as waveguide optics, nondestructive testing, and remote sensing.

Here we focus our attention to the resonances of transmission problem, which has importance in non-destructive testing of anisotropic materials, in the whole of $\mathbb{R}^n$, $n = 2, 3$, where the conductivity is assumed to take a positive constant value outside the domain Ω. The transmission eigenvalue problem is a nonlinear and non-selfadjoint eigenvalue problem that is not covered by the standard theory of eigenvalue problems for elliptic equations. The eigenvalue problem with the Dirichlet boundary condition (the Neumann problem, in isotropic media, was treated in [4]), and the scattering problem in $\mathbb{R}^n \setminus \Omega$ with the Dirichlet or the Neumann boundary condition are of equal interest. The asymptotic results for the eigenvalues or the resonances in such cases can be obtained with only minor modifications of the techniques presented here and in [4,5,13,20], while the rigorous derivations of similar asymptotic formulae for the scattering problems for the electromagnetic waves (full Maxwell's equations) or for the Stokes equations require further works.

This work is considerably different from that in [1,2,4,5,8,9,13,20] for the eigenvalue problem. In [2,20], we have combined the expansions derived in [11] with a theorem developed by Osborn [24] about the convergence of eigenvalues of some compact operators.

The novelty of this paper, is that it leads to analysis of resonance problems in the presence of multiple anisotropic imperfections. By referring to previous works, the resonance problem is more difficult because the resonance values are not the eigenvalues of a set of compact operators. They can, however, be viewed as singular values of a sequence of meromorphic operator functions. This is achieved by rewriting the problem in terms of integral equations on the boundary of the domain. Then, by using complex operator theory, we derive a formula for the convergence of the resonance values which is in the spirit of the theorem of Osborn. This then leads to an asymptotic expansion for the resonances which is similar to that for the eigenvalues.

The leading order term in the asymptotic expansion contains information about the location, shape, and material properties of the imperfections.

The paper is organized as follows. In Section 2, we describe the geometry of the domain with its constitutive parameters and we formulate the perturbed resonance problem associated with the Laplacian operator (2.3). In Section 3, we reformulate the problem (2.3) and its unperturbed configuration (2.4) as two systems of integral equations depending on the small parameter ϵ which is the scale factor associated to anisotropic imperfections. By the analytic Fredholm theory, we transform these systems into the determination of the poles of two meromorphically continued inverse integral operator-valued functions $T^{-1}(w)$ and $T_\epsilon^{-1}(w)$ in the complex plane. Section 4 is devoted to the main results of this paper such as Theorems 1 and 2.

2. Problem formulation

In this paper we consider the bounded domain $\Omega \subset \mathbb{R}^n$ ($n = 2$ or 3) with C^2-smooth boundary $\partial\Omega$, and let ν denote the outward unit normal vector on $\partial\Omega$. We suppose that Ω contains a finite number m of (possibly anisotropic) bounded imperfections (see Fig. 1). The geometry of each one is of the form $D_i = z_i + \epsilon B_i$, with a smooth and simply connected boundary ∂D_i, where $B_i \subset \mathbb{R}^n$ is a regular enough bounded domain representing the volume of the imperfection,

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