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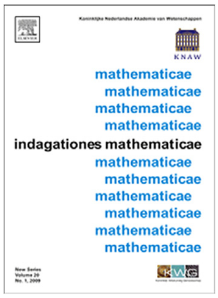

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Some Qualitative properties of Finsler p-Laplacian

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Abstract

The Finsler p-Laplacian is the class of nonlinear differential operators given by

$$\Delta_{H,p}u := div(H(\nabla u)^{p-1}\nabla_\eta H(\nabla u))$$

where $p > 1$, $H : \mathbb{R}^n \to [0,\infty)$ is a convex function which is in $C^1(\mathbb{R}^n\backslash\{0\})$ and is positively homogeneous of degree 1. In this article we provide a comparison principle, weighted Poincare Inequality, Liouville Theorem and Hardy type inequality for the Finsler p-Laplacian.

Keywords: Finsler p-Laplacian, Picone Identity, Hardy Inequality, Comparison Principle

1. Introduction

The main purpose of this work is to provide some results on qualitative properties of the generalised class of differential operators called the anisotropic p-Laplacian or the Finsler p-Laplacian and is given by

$$\Delta_{H,p}u := div(H(\nabla u)^{p-1}\nabla_\eta H(\nabla u)) \tag{1}$$

where $p > 1$, $H : \mathbb{R}^n \to [0,\infty)$ is a convex function which is in $\mathbf{C}^1(\mathbb{R}^n\backslash\{0\})$ and is positively homogeneous of degree 1, where ∇ and ∇_η are gradient operators for x and η respectively.

If we consider the function $H(x) = ||x||_q = \left(\sum_1^n |x_i|^q\right)^{\frac{1}{q}}$ for $q > 1$ then we have,

$$\Delta_{p,q}u := div\left(||u||_q^{p-q}\nabla^q u\right) \tag{2}$$

where,

$$\nabla^q u = \left(|u_{x_1}|^{q-2}u_{x_1}, \cdot, \cdot, \cdot, |u_{x_n}|^{q-2}u_{x_n}\right) \tag{3}$$

Putting $q = 2$ and $p \in (1,\infty)$ in (3) we have the p-Laplacian operator, $p = q > 1$ gives us the pseudo p-Laplacian and $p = q = 2$ reduces (3) to the ordinary laplacian. Our aim in this note is to generalize some results available for the p-Laplacian to the Anisotropic case such as the Liouville Theorem, Hardy Inequality, Comparison Principle for sub and super solution and weighted Poincaré Inequality. We will start by generalizing the Picone Identity of Jaros [11] in the nonlinear setting as in Bal [3] and then use it to study various properties related to the anisotropic p-Laplace operator.

2. Preliminaries

We begin this section with some properties of an arbitrary norm in $\mathbb{R}^n$. Let H be any arbitrary norm in $\mathbb{R}^n$ i.e, a convex function from $\mathbb{R}^n$ to $[0,\infty)$ such that

1. $H(\eta) > 0$ for any $\eta \neq 0$.
2. $H(t\eta) = |t|H(\eta)$ for all $\eta \in \mathbb{R}^n$ and $t \in \mathbb{R}$.
3. If H is $C^1(\mathbb{R}^n\backslash\{0\})$ then $\nabla_\eta H(t\eta) = \text{sgn}\, t\, \nabla_\eta H(\eta)$ for all $\eta \neq 0$ and $t \neq 0$.
4. $\langle \eta, \nabla_\eta H(\eta)\rangle = H(\eta)$ for all $\eta \in \mathbb{R}^n$ where the left expression is zero for $\eta = 0$.
5. There exists constants $0 < c_1 \leq c_2$ such that $c_1|x| \leq H(x) \leq c_2|x|$.