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An implicit time-stepping scheme and an improved contact model for ice-structure interaction simulations

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ABSTRACT

This paper presents a novel time-stepping scheme for the modelling of discrete ice-structure interaction. The scheme extends the *non-smooth* discrete element modelling (NDEM) technique to enable compliant continuous and discontinuous contacts. This increases the accuracy and expands the applicability range of the NDEM technique related to ice-structure interaction problems. We derive the parameters representing the compliant behaviour of contacts. The accuracy of the presented scheme for discontinuous contacts is compared to an existing, simpler scheme that limits the contact force based on a maximum force assumption. The comparison shows that the derived scheme results in more accurate contact forces, for the same time step size, as previously applied NDEM schemes in ice-structure interaction. An example simulation is compared against ice tank tests of a 4-legged, vertical-walled structure moving through a broken ice field.

1. Introduction

The reduction of the areal extent and thinning of the Arctic sea ice cover will increase activity in waters where sea ice may occur. The accurate prediction of the loads and resistance caused by sea ice is important for safe and economical operations in these waters. Existing calculation methods for loads from sea ice on structures often rely on empirical formulas based on a limited range of full-scale data. Full-scale data are limited to existing structures and the regions where they are located. On top of this, the data are often incomplete and there is a high uncertainty in the measured loads. Ice tank tests can be used to obtain load data for specific types of structures or ice conditions. However, it is often uncertain if and how the loads measured in the ice tank can be scaled to full-scale equivalent loads. This is especially challenging for load cases other than continuous level ice, such as floe ice or ice ridges. Numerical modelling of ice-structure interaction can help, in combination with full-scale and model test data, to increase the understanding of occurring phenomena and ice failure modes, and can lead to a more accurate prediction of the sea ice loads that may be encountered.

Interaction between sea ice and structures is a complicated process. There are many factors that may contribute to the load and resistance experienced by a structure interacting with sea ice, and that pose challenges to the accurate numerical representation of the occurring processes. For example:

- many simultaneously contacting ice bodies
- complicated, and (seemingly) random body geometries
- difficult to estimate and highly variable ice material properties
- complicated hydrodynamic effects
- complicated and continuous dynamic fractures and failures

This combination of factors makes ice-structure interaction different from any other engineering problem. A numerical model will need to simplify some or all of the above-mentioned factors. To what extent the parameters can (and need to) be simplified depends on the processes to be investigated and limiting factors such as the available computing power and computation time, as well as the availability and accuracy of input parameters. The broad range of occurring processes and the different requirements that may be put upon numerical models, has led to a broad range of models and modelling types.

Numerical models for sea ice load estimation can broadly be divided in continuum and discrete models, although there are also several models that combine both modelling types. Among discrete ice-structure interaction models, a further distinction can be made based on the time-stepping scheme that is used. This difference is often described as *smooth* discrete element modelling (SDEM) versus *non-smooth* discrete element modelling (NDEM). The difference between NDEM and SDEM can be seen as the difference between implicit and explicit time integration (Servin et al. (2014)), allowing for much larger time steps, while maintaining stable simulations, when using NDEM. The time

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steps can often be several orders of magnitude larger than those in SDEM, but the processing of each time step is more computationally expensive. NDEM requires the solution of a linear complementarity problem (LCP) or a mixed linear complementarity problem (MLCP) at each time step. Nevertheless, NDEM simulations are often considered more efficient and are mostly chosen when real-time or near-real-time simulations are required.

There are many publications describing SDEM models and modelling results of ice-structure interaction, going back to the early 90s. Some examples can be found in Hopkins et al. (1991); Hocking (1992); Løset (1994*a*,b); Tuhkuri and Polojärvi (2005); Polojärvi and Tuhkuri (2009); Paavilainen et al. (2009); Liu et al. (2017). The application of NDEM in ice-structure interaction has been more recent, and so far it has been mostly applied to global broken ice-structure simulations. Application examples of NDEM in ice-structure interaction can be found in Konno and Mizuki (2006); Lubbad and Løset (2011); Metrikin (2014); Alawneh et al. (2015); Yulmetov et al. (2016). In NDEM, contacts between interacting bodies are often assumed to be infinitely rigid. Therefore, contact forces cannot be defined in a physically correct manner. This limitation can be remedied to some extent by introducing an upper limit for the contact force based on a combination of contact area and crushing pressure, as is done in Lubbad and Løset (2011) and Metrikin (2014), where both papers use a slightly different method to apply the upper limit.

In this paper, we derive a novel NDEM time-stepping scheme starting from the Newmark-Beta method for differential equations (Newmark (1959)). The new formulations are valid for compliant continuous and discontinuous contacts. The position and velocity update rules of the Newmark-Beta method are rewritten, and limits are introduced in order to enable discontinuous contact modelling. Compared to previously applied NDEM methods in ice-structure interaction modelling, where only an upper limit for the contact force was defined based on the current contact area, our new method takes the current contact area as well as the expected change in the contact area into account in determining the contact response, leading to a higher accuracy of the predicted contact force for the same time step size. The main properties of the NDEM time-stepping scheme are maintained in the new method, i.e., an MLCP is solved in each time step, and large time steps can be taken without affecting the stability. In addition, the new method can now handle compliant as well as infinitely stiff contacts. The method is implemented in the *Simulator of Arctic Marine Structures* (SAMS), the product of *Arctic Integrated Solutions* (ArcISo); see *ArcISo* (2018); Lubbad et al. (2018).

To the authors' knowledge, this is the first time such a time-stepping scheme has been applied to ice-structure interaction modelling. In other fields, such as soil modelling and physics simulations, similar methods have been described and used Jean (1999); Moreau (1999); Lacoursière (2007); Krabbenhoft et al. (2012); Tasora et al. (2013); Servin et al. (2014). The difference between these methods and the method described in this paper is that the current model assures energy conservation for continuous linear contacts. This is a property of the Newmark-Beta method. The other methods are derived from an implicit Euler or similar schemes, and therefore result in numerical damping.

In Section 2, we first derive a generalized form of the time-stepping scheme that applies to rigid, compliant, continuous and discontinuous contacts. Sections 3 and 4 describe how the needed contact parameters can be obtained for continuous contacts, as would occur in a lattice model, and discontinuous contacts, such as ice-ice and ice-structure contacts. The accuracy of the derived scheme for discontinuous contacts is compared against an existing scheme in Section 5. In Section 6 we provide an application example, in which we compare the results from the numerical model against data obtained in an ice tank test. Section 7 discusses some features of the numerical model and the application example. Finally, Section 8 concludes the paper.

2. An implicit DEM time-stepping scheme

The proposed implicit time stepping-scheme expands the traditional NDEM formulation to include compliant contact behaviour, which is needed for the accurate simulation of ice-structure interactions. Similar to the traditional NDEM formulation, the stability of the simulations is independent of the time step size when using the proposed scheme, and it is capable of efficiently solving a large network of simultaneous contacts.

The following sections derive the MLCP, which needs to be solved at each time step. The central assumption in the derivations is a constant average acceleration within a time step. More particularly, this means that we use the average force occurring within a time step in body propagation. It does not mean, however, that the contact force itself is assumed constant. This corresponds to a Newmark-Beta method (Newmark (1959)) with parameters $\gamma = \frac{1}{2}$ and $\beta = \frac{1}{4}$, yielding the constant average acceleration method. We start by deriving some terms for a continuous 1 degree of freedom (DOF) case, then add constraints to the contact force to enable discontinuous contact modelling, and finally compare the resulting formulation to previously used formulations. The expansion to multiple degrees of freedom and frictional contacts is explained in Appendix B, since this part is similar to previously applied methods.

2.1. Derivation of the time-stepping scheme for a continuous 1-DOF case

Fig. 1 shows the 1-DOF example case used for the derivations in this section. In this example case, we use a generalized Kelvin-Voigt unit as the contact model, in which the parallel spring and dashpot element can be linear or nonlinear. The method can also be applied to other rheological elements, such as a Maxwell unit, following a similar procedure as described in this section. In Fig. 1, m stands for the mass of the body, δ for the penetration depth, $\dot{\delta}$ for the penetration velocity, $F_{\text{cont}}(\delta, \dot{\delta})$ for the contact force as a function of the penetration and the penetration velocity, and F_{ext} stands for an external (non-contact) force acting on the body during time step $\Delta t = t_{n+1} - t_n$, where t_n is the current time and t_{n+1} is the time at the end of the time step. u, $\dot{u}$ and $\ddot{u}$ are the body position, velocity and acceleration, respectively. For convenience, we choose the axis system such that $\delta = u$ if $\delta \geq 0$. In the derivation in this section, we assume $\delta \geq 0$, and thus $F_{\text{cont}}(\delta, \dot{\delta}) = F_{\text{cont}}(u, \dot{u})$. This is expanded to a case in which $u \in \mathbb{R}$ in Section 2.2. The equation of motion of this system is:

$$m\ddot{u} + F_{\text{cont}}(u, \dot{u}) = F_{\text{ext}} \tag{1}$$

Assuming constant average acceleration within each time step, the equation of motion can be discretized, and body positions and velocities are updated according to Eqs. (2) and (3), which are the time-stepping equations as used in the constant average acceleration method:

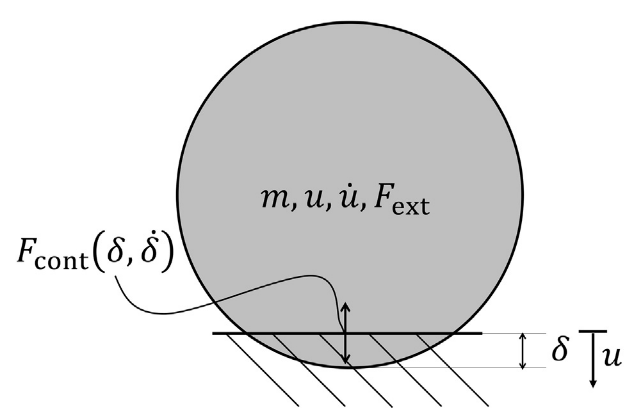

Fig. 1. General single DOF contact case.