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# A resistivity model for testing unfrozen water content of frozen soil

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ABSTRACT

Testing the unfrozen water content ($\theta_u$) based on frozen soil resistivity (ρ) is considered to be a common and convenient method. However, the reliability of the theoretical model for testing unfrozen water content requires further study; hence this paper proposes a new resistivity model for testing the unfrozen water content. By considering the variable characteristics of unfrozen water in the soil freezing process and based on the three-element electrical conduction model, four types of micro unit representing different unfrozen water contents were proposed, and the resistivity model was established on the basis of microscopic analysis. The resistivity model showed that a power function can be used to describe the relationship between the soil resistivity and unfrozen water content of frozen soil. Silty clay was used as a research object. The temperature, unfrozen water content and resistivity of soil samples with different initial water content and same soil density were tested by Nuclear Magnetic Resonance (NMR) technology and resistivity testing equipment. The results of the experiments performed on frozen silty clay verified the reasonableness of the proposed model for the resistivity of frozen soils. The conductive mechanism of frozen soil was discussed, as well as the resistivity model and its in-situ application for unfrozen water testing.

## 1. Introduction

Unfrozen water content is an important index and it has critical influence on the strength and deformation characteristics of frozen soil (Xu et al., 2017; Liu et al., 2017; Xu et al., 2016a,b, 2015; Lai et al., 2014). A series of methods for determining unfrozen water content of frozen soil were put forward in laboratory such as Nuclear Magnetic Resonance (NMR), Time Domain Reflectometry (TDR), Calorimetry and ultrasonic wave velocity (Watanabe and Wake, 2009; He et al., 2016; Kozlowski, 2016; Wang et al., 2003; Liu and Li, 2012), etc. The methods of laboratory testing of unfrozen water content of frozen soil were relatively accurate (Yoshikawa and Overduin, 2005). Past studies (Liu et al., 2014; Tang et al., 2005a,b) have been some research for the field testing techniques of unfrozen water content, however, the accuracy of the test results needs to be improved. It is only possible to roughly determine the location of the upper boundary of the permafrost and test the approximate value of the unfrozen water content. Kojima et al. (2014) used the method of Sensible Heat Balance to determine soil ice contents and the in situ ice contents determined with the SHB method were sometimes unrealistically large or even negative. Suitable methods of in-situ testing of unfrozen water content have been rarely discussed. Based on the link between unfrozen water content and resistivity of frozen soil, many scholars tried to study the distribution and content of water in soil by the change of soil resistivity.

In an orthogonal test of soil resistivity, it was demonstrated that the factors affecting soil resistivity were water content (primary factor) as well as conductivity of pore water, saturation and soil type (secondary factors) (Liu et al., 2004a,b). Kalinski and Kelly (1993) studied the relationship between soil resistivity and water content and concluded that the volumetric water content could be estimated from soil resistivity within a standard error of 0.009. Pan et al. (2016) made a study on the relationship between frozen soil temperature and resistivity. The results showed that when temperature diminished significantly, the pore water content in frozen soil dropped and the soil resistivity increased when temperature diminished significantly. Oldenborger and LeBlanc (2017) used the high density resistivity detection technology for unfrozen water content, and obtained more reliable data. Based on the laboratory test, Tang et al. (2017a,b) established the relationship among soil resistivity, temperature and unfrozen water content. A set of in-site unfrozen water content testing equipment was developed based on the above relationship. Although the monitoring of in-situ unfrozen water content via soil resistivity was carried out, the lack of a theoretical guide, i.e. a scientific model of frozen soil resistivity, lead to inaccurate results in the moisture measurement.

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Cai et al. (2017) reviewed the conductivity model of saturated porous media and summarized the conductive mechanism and conductive model of porous saturated medium. Archie (1942) carried on the research to the soil resistivity model, and obtained the resistivity computational model for saturated soil. Waxman and Smits (1968) proposed a resistivity model for unsaturated cohesive soil. Rhoades et al. (1989) found that there are three conductive paths in the soil, and that the conductive path is formed by the coupling of soil and water. A schematic diagram of three-element electrical conduction model was presented by Mitchell and Soga (1993), and the resistivity formula of cohesive soil was deduced by (Zha et al., 2006) on the basis of three conductive paths (Rhoades et al., 1989). Based on the test data in Umiujaq, Nunavik Canada (1994) and theoretical deduction (2008), the resistivity model can be applied to frozen soil and unfrozen soil was proposed by (Fortier et al.) In the study of permafrost resistivity, Harada and Fukuda (2009) improved Archie's model. Results showed that the model can be applied to the soil with high water content, high clay content and temperature close to 0 °C. Based on the three-element electrical conduction model (Mitchell and Soga, 1993), Shan et al. (2015) established the micro resistivity model and verified the rationality of that model via experimental data. However, it was not possible to explain the variation of resistivity at the macro level by the micro resistivity model which Shan et al. (2015) set up.

In conclusion, taking the unique physical properties of frozen soil into account, the variation of frozen soil resistivity is reflected by a micro unit model at the micro level, which cannot represent the variation of the frozen soil resistivity with the inhomogeneity of moisture distribution at the macro level. Therefore, it is necessary to consider the impact of moisture migration on the change of frozen soil resistivity at the macro level.

In this paper, four types of resistivity models representing different unfrozen water contents at the micro level were proposed. These different types were then integrated with the macro soil resistivity by implementing the law of moisture migration during soil freezing as well as by obtaining the macro model of the frozen soil resistivity. Specifically, the relationship between resistivity and unfrozen water content was tested by NMR equipment, and the macro model of the frozen soil resistivity was verified. Finally, the application of frozen soil resistivity model was discussed in the unfrozen water content measurement.

## 2. Establishment of theoretical model for resistivity of frozen soil

During the soil freezing process, the distribution of unfrozen water in frozen soil is inhomogeneous due to the water migration. A micro unit model could not fully represent the frozen soil resistivity change at the macro level. Therefore, four types of micro units representing different unfrozen water contents were proposed and the micro unit resistance was deduced from these types. The freezing process of soil was composed by freezing prophase, freezing metaphase and freezing anaphase. Afterwards, the resistivity model at the macro level of each freezing phase was obtained by integral method. The resistivity variation trend of each freezing phase was analyzed, and the resistivity model of frozen soil in the whole freezing process was proposed.

### 2.1. The resistance of micro unit

Based on the three-element electrical conduction model of Mitchell (Mitchell and Soga, 1993), four types of micro units representing different unfrozen water contents are proposed for solving the resistivity model of frozen soil. The micro unit *a* represents the state of moisture in the soil that has not yet been frozen, micro unit *b* represents the state of free water starting to freeze in the soil, micro unit *c* demonstrates the state of completely frozen free water in the soil, and micro unit *d* shows the state of the bound water starting to freeze in the soil (Fig. 1).

The micro units are assumed to be 1 cube ($L = 1$), the direction of

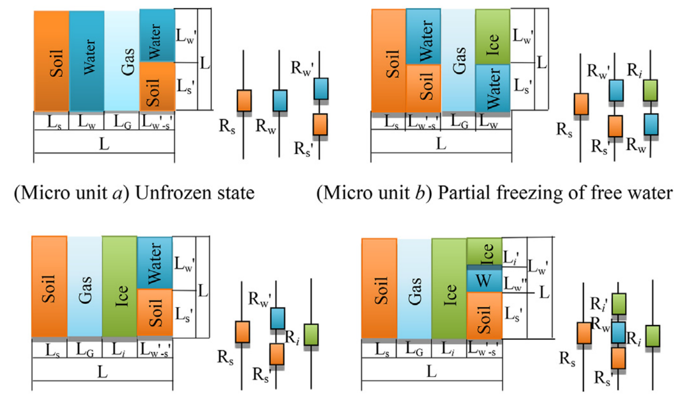


**Fig. 1.** Micro unit representing different unfrozen water content (*Ls* and *Ls′* represent the widths of the parallel and series paths, respectively, that are composed of soil particles; *Lw* and *Lw′* represent the widths of the parallel and series paths, respectively, that are composed of free water and bound water; *Li* and *Li′* represent the widths of the parallel and series paths, respectively, that are composed of ice; *LG* represent the widths of the parallel and series paths, *Rs* and *Rs′* represent the resistivity values of the soil particles in the parallel and series paths, respectively; *Rw* and *Rw′* represent the resistivity values of the free water in the parallel and the bound water in series paths, respectively; *Ri* and *Ri′* represent the resistivity values of the ice in the parallel and series paths, respectively).

the current was vertical, and the conduction of the gas in the soil is ignored. The resistivity of soil, water, ice, are expressed as $\rho_s$, $\rho_w$, $\rho_i$ respectively. The coefficient of soil conductivity, which is the ratio between the width of soil water series and the length of the entire soil body, is denoted by $F'$ ($F' = L_{w'-s}'/L = L_{w'-s}'$). The porosity of soil is denoted by $n$. The resistivity models of 4 types of micro unit are deduced respectively.

Micro unit *a* (Fig. 1*a*) represents the distribution of unfrozen water under unfrozen condition. The volume water content of the parallel part of soil water is denoted by $N$ ($N = L_w/L_s$), the volume water content of the series of soil water is denoted by $M$ ($M = L_w'/L_s'$). The saturation of micro unit *a* is denoted by $S_{r1}$, then:

$$L_s + L_w + L_a = 1 - F' \tag{1}$$

$$L_w' + L_s' = L = 1 \tag{2}$$

$$L_w + L_G + F' \cdot L_w' = n \tag{3}$$

$$\frac{(L_w + F' \cdot L_w')}{n} = S_{r1} \tag{4}$$

$$\frac{L_w'}{L_s'} = M \tag{5}$$

$$\frac{L_w}{L_s} = N \tag{6}$$

From Eqs. (1)–(6), the following can be obtained:

$$L_w = \frac{nS_{r1}(M+1) - F'M}{M+1} \tag{7}$$

$$L_w' = \frac{M}{M+1} \tag{8}$$

$$L_s' = \frac{1}{M+1} \tag{9}$$

$$L_s = \frac{nS_{r1}(M+1) - F'M}{(M+1)N} \tag{10}$$

$$L_G = n - \frac{(M+1)nS_{r1} - 2F'M}{(M+1)} \tag{11}$$

If the total resistance of micro unit *a* is $R_a$, then: