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Mathematical solution of steady-state temperature field of circular frozen wall by single-circle-piped freezing

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ABSTRACT

Ground temperature distribution is one of the fundamental problems in the theory of artificial ground freezing. For the most widely applied freezing scheme, single-circle-piped freezing, there is only one analytical solution up to now, for the situation that the core is totally frozen. This paper gives a mathematical model to the steady-state temperature field of circular frozen wall by single-circle-piped freezing with unfrozen core. Then, the solutions are theoretical derived with some hypothesis, using conformal mapping and boundary separation method, as well, the same methods has been verified by the specific single-row-piped solution – Bakholdin's solution. After that, comparison between the analytical solutions and the numerical analysis is in good consistency. In addition, the methods to calculate the thickness and the average temperature of circular frozen wall are present.

1. Introduction

The first recorded application of artificial ground freezing (AGF) was on a mineshaft in Swansea, South Wales circa 1862, although the method was patented by Germany engineer F. H. Poetsch in 1883. At present, AGF technique is a mature construction method, which is widely used in fields like mine engineering, tunneling engineering, remediation of subsurface contaminants, and ground source heat pump, etc. (Vitel et al., 2016; Marwan et al., 2016; Wagner, 2013; Zheng et al., 2016).

For structure design and construction management as well as risk control of freezing projects, some engineering parameters, such as the thickness and the average temperature of the frozen wall, are of great significance (Jiang et al., 2012). Hence, both for designers and for constructors, it is necessary to master the temperature distribution and its developments. Among varied methods, the analytical solution based on temperature distribution models meet with great favor due to utility.

Some analytical solutions of steady-state temperature field in AGF have been derived since the middle of last century. In Russia, Trupak (1954) proposed the calculation methods to a single-pipe freezing and single-row-piped freezing which is simple numeral superposition by single-pipe's solution. Compared the thermal problems with hydraulic ones, Bakholdin (1963) proposed the analytical solutions to single-row- and double-row- piped freezing. Sanger and Sayles (1979) divided the freezing process into three stages, and deduced thermal calculations for each stage. Tobe and Akimoto (1979) derived an analytical solution to temperature field of multi-piped freezing. Hu et al. (2008a), Hu et al. (2008b), Hu and Zhao (2010), Hu (2010), Hu et al. (2011), Hu et al. (2012) refined the above-mentioned solutions and studied their applications. By means of superposition of potential (Pollack and Stump), Hu's team has pushed forward with the studies. Hu et al. (2013) presented single-circle-pipe freezing with frozen core. Then, Hu et al. (2016) gave the steady-state solutions of multi-piped freezing while Hu et al. (2017) derived the solutions to three-row-pipe freezing.

In practice, circle-piped freezing is the oldest, the most classical, but the most widely used type in AGF, especially for mine shaft sinking. However, for more than 150 years since the first application of the innovation in 1862, there has been no solution for the temperature field of single-circle-piped freezing with unfrozen core. To solve this centurial problem, this paper gives an analytical solution, using conformal mapping and boundary separation method for harmonic equations. Then, applications of the solution have been derived.

2. Analytical theory

2.1. Steady-state conduction

The original three-dimensional heat conduction problem can be simplified as a two-dimensional one in a real AGF project in that the temperature gradient along the freezing pipe is gentle (Pimentela et al.,

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Notation

k	thermal conductivity
l	distance between two freezing pipes
n	number of freezing pipes
R	polar radius in Z-plane
R_0	radius of freezing pipe
R_1	polar radius of the inner boundary of frozen soil wall
R_2	polar radius of the freezing-pipe circle
R_3	polar radius of the outer boundary of frozen soil wall in the master section
R_3'	polar radius of the outer boundary of frozen soil wall in the interface section
R_w	radius of freezing pipe in ζ-plane
t	time
T	distribution of temperature field
T_0	temperature at the boundaries of the frozen soil, i.e. the freezing temperature of the soil
T_{cp}	average temperature of Bakholdin's solution
$T_{avg\xi}$	average temperature of the frozen wall in ζ-plane
T_{avgZ}	average temperature of the frozen wall in Z-plane
T_f	temperature of the freezing pipes
u, v	coordinate in ζ-plane
x, y	coordinate in Z-plane
Z	object plane
θ	polar angle
ξ	thickness of the straight frozen soil wall in the master section
ξ'	thickness of the straight frozen soil wall in the interface section
ξ_1	thickness of frozen wall on the inner side in ζ-plane
ξ_2	thickness of frozen wall on the outer side in ζ-plane
ζ	image plane

2012). According to Carslaw & Jaeger, the temperature field caused by conduction is mainly discussed here as the other two (radiation and convection) do not contribute significantly to the formation of temperature field (Carslaw and Jaeger, 1959). Then, by the first law of Fourier heat conduction (Latif, 2009), the equation of two-dimensional steady-state heat conduction is:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) = 0 \tag{1}$$

where k is the coefficient of thermal conductivity of soil, T the distribution of temperature field, and x, y coordinates, respectively.

We assume that the soil is isotropic in thermal physics, so k is identical in all directions. In addition, k is the same once the temperature drops below the freezing point (this assumption approximates to the real case except when the phase change of soil occurs). Then, (1) can be simplified as follows:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{2}$$

whose polar form can be written as:

$$\frac{\partial^2 T}{\partial R^2} + \frac{1}{R}\frac{\partial T}{\partial R} + \frac{1}{R^2}\frac{\partial^2 T}{\partial \theta^2} = 0 \tag{3}$$

Now the original engineering problem has been converted to the solving of a plain Laplace equation under specified boundaries.

2.2. Model of single-pipe freezing

The model of single-pipe freezing in infinite field is shown in Fig. 1. Its mathematical model is as follows:

$$\begin{cases} \frac{\partial^2 T}{\partial R^2} + \frac{1}{R}\frac{\partial T}{\partial R} + \frac{1}{R^2}\frac{\partial^2 T}{\partial \theta^2} = 0 \\ T(R_0, \theta) = T_f; \text{the condition of freezing pipe} \\ T(\xi, \theta) = T_0; \text{the boundary conditon of frozen soil} \end{cases} \tag{4}$$

The general solution to the above harmonic equation obtained using the method of separation of variable is given (Chen et al., 2003) below:

$$T(R,\theta) = c_{10} + c_{20}\ln R + \sum_{m=1}^{\infty}(a_m\cos m\theta + b_m\sin m\theta)(c_{1m}R^m + c_{2m}R^{-m}) \tag{5}$$

where c_{10}, c_{20}, a_m, b_m, c_{1m}, c_{2m} are undetermined coefficients which are calculated by the boundary condition.

Expand the boundary conditions according to the Fourier series and constants not dependent to θ where $a_m = 0$ and $b_m = 0$. Then introducing the boundary conditions of the freeze pipes and frozen soil into Eq. (5) gives:

$$T = T_f\frac{\ln\frac{\xi}{R}}{\ln\frac{\xi}{R_0}} + T_0\frac{\ln\frac{R}{R_0}}{\ln\frac{\xi}{R_0}} \tag{6}$$

This is Trupak's solution to the temperature field of single-piped freezing (Trupak, 1954).

3. Single-circle-piped freezing with unfrozen core

3.1. Mathematical model

The boundary of the frozen ground appears wavy in shape after the closure of frozen soil columns, so the model of single-circle-pipe freezing with unfrozen core is shown in Fig. 2.

The mathematical expressions of the model are:

$$\begin{cases} \frac{\partial^2 T}{\partial R^2} + \frac{1}{R}\frac{\partial T}{\partial R} + \frac{1}{R^2}\frac{\partial^2 T}{\partial \theta^2} = 0 \\ T\left(R_2 + R_0, j\frac{2\pi}{n}\right) = T_f; \text{the condition of freezing pipes} \\ T\left(R_1, j\frac{2\pi}{n}\right) = T_0; \text{the inner boundary condition of the frozen soil} \\ T\left(R_3, j\frac{2\pi}{n}\right) = T_0; \text{the outer boundary conditon of the frozen soil} \end{cases} \tag{7}$$

where n is the total number of freezing pipes and j covers the integers between 0 and n-1.

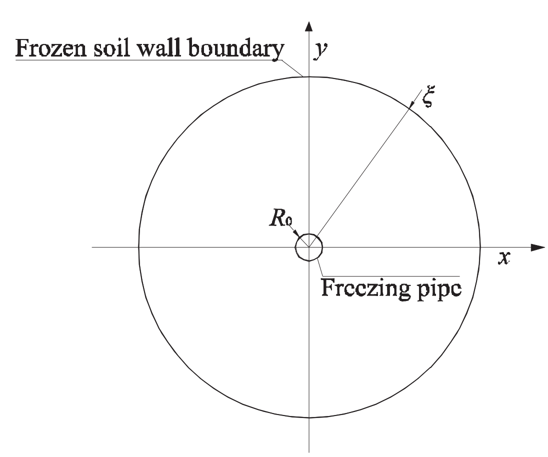

Fig. 1. Model of single-pipe freezing.