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A statistical damage constitutive model under freeze-thaw and loading for rock and its engineering application

Shibing Huang[a,b,c,*], Quansheng Liu[c], Aiping Cheng[a,b], Yanzhang Liu[a,b]

[a] School of Resources and Environmental Engineering, Wuhan University of Science and Technology, Wuhan, Hubei 430081, China
[b] Hubei Key Laboratory for Efficient Utilization and Agglomeration of Metallurgic Mineral Resources, Wuhan, 430081, China
[c] State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan, Hubei 430071, China

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ABSTRACT

The freeze-thaw damage of rock is a critical problem to study rock engineering in cold regions. When water is frozen in pores, 9% volume expansion will induce damage in rock for a huge ice pressure. Thus, rock will be deteriorated and softened after freeze-thaw. Besides, loading damage produces when the inner stress exceeds the bearing capacity of rock. So it's crucial to establish a damage constitutive model under freeze-thaw and loading in order to evaluate the stability of rock engineering in cold regions. The freeze-thaw damage of rock is expressed by static elastic modulus in this paper, which can be accurately and simply predicted by damage evolution equation using P-wave velocity or compression strength proposed by Liu et al., 2015. Loading damage can be obtained by statistical theory assuming that micro-unit strength satisfies the Weibull distribution and the maximum-tensile-strain yield criterion. Then the final statistical damage constitutive equation under freeze-thaw and loading is derived, in which unknown parameters can be determined through carrying out compression experiment on rock after different freeze-thaw cycles. This developed model is validated by a previous experiment and applied to analyze the stability of a tunnel under coupled thermo-hydro-mechanical condition in cold regions. The results show that the proposed constitutive model is very suitable for rock suffering freeze-thaw cycles and it is of high accuracy and good practicality.

1. Introduction

The high speed trains, designed speed of which is more than 400 km/h, will bring cold air into tunnel, so not only the surrounding soil in the exit and entrance but also the surrounding rock and lining structure in the middle of tunnel will be damaged under freeze-thaw in cold regions which seriously threats the operation security (Tan et al., 2014). As a result, the freeze-thaw damage and failure of rock was usually observed in cold region tunnels (Zhang et al., 2004; Park et al., 2010). Actually, freeze-thaw damage is mainly caused by repeated initiation and dissipation of pore ice pressure due to 9% volume expansion of freezing water in pores (Park et al., 2014; Freire-Lista et al., 2015). Thus, the freeze-thaw action of rock may be regard as fatigue damage (Liu et al., 2015).

With more and more freeze-thaw damage problems of rock engineering arising, it has aroused the research enthusiasm of scholars. However, the previous studies are focused on freeze-thaw tests and freeze-thaw damage models. In order to research the evolution rule of freeze-thaw damage in rock, many rock parameters are measured during freeze-thaw cycles including porosity, P-wave velocity, mass, uniaxial compressive strength and so on (Matsuoka, 1990; Takarli et al., 2008; Luo et al., 2014; Momeni et al., 2016). Combined with those characteristic parameters, a series of damage models were proposed to study the freeze-thaw process (Mutlutürk et al., 2004; Yavuz et al., 2006; Jamshidi et al., 2013; Liu et al., 2015; Wang et al., 2016). Besides, deterioration due to freeze-thaw action can also affect the stiffness of rock (Tan et al., 2011). Thus, the stress-strain relationship of freeze-thaw damaged rock significantly changes. This constitutive relationship under different freeze-thaw cycles should be firstly established in order to evaluate the stability of rock engineering in cold regions. However, generally what we can obtain are the stress-strain curves after several fixed freeze-thaw cycles in laboratory and the frost damage of rock engineering in cold regions is induced by the coupling action of freeze-thaw and loading. So it's a crucial subject to build the damage constitutive model under freeze-thaw and loading after any cycle according to a few available information.

Considering the shortages of previous research, in this paper, firstly we deduced a statistical constitutive equation under freeze-thaw and loading in which the freeze-thaw damage is reflected in the change of static elastic modulus. Then the evolution rule of static elastic modulus

* Corresponding author at: School of Resources and Environmental Engineering, Wuhan University of Science and Technology, Wuhan, Hubei 430081, China.
E-mail address: huangshibing@wust.edu.cn (S. Huang).

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for damage rock under freeze-thaw was studied and the unknown parameters of this constitutive model could be determined by fitting the experimental data. In Part 3, the developed model had been validated by a representative experiment conducted by Tan et al. (2011). In Part 4, a stability analysis of cold-regional tunnel after different freeze-thaw cycles was done under coupled thermo-hydro-mechanical (T-H-M) condition by implementing the proposed constitutive model into finite element software. Finally, several significant conclusions have been drawn from this study.

2. Establishment of the statistical damage constitutive model

2.1. Statistical damage constitutive equation under freeze-thaw and loading

The damage of rock in cold regions consists of freeze-thaw damage and loading damage. The freeze-thaw damage of rock is reflected in the change of static elastic modulus. So firstly we deduced a damage constitutive equation under loading based on statistical theory of micro unit strength. Then replacing the initial static elastic modulus with damaged static elastic modulus after freeze-thaw, the final damage constitutive equation can be derived.

According to the stress equivalence principle from Lemaitre (1985), the effective stress in fresh rock under loading can be expressed as

$$\overline{\sigma}_i = \frac{\sigma_i}{1 - D_l} \tag{1}$$

where σ_i is the nominal principal stress, $\overline{\sigma}_i$ is the effective principle stress of damage rock under loading, D_l is damage variable under loading, $i = 1, 2, 3$.

There is a lot of micro pores and fissures in rock, but we cannot take full consideration of those micro defects. So the micro unit strength of rock is assumed to satisfy certain statistical distribution. Assuming that the micro unit strength of rock satisfies Weibull distribution, the probability density function of micro unit strength is

$$P(\overline{f}\,) = \frac{m}{f_0}\left(\frac{\overline{f}}{f_0}\right)^{m-1} \exp\left[-\left(\frac{\overline{f}}{f_0}\right)^m\right] \tag{2}$$

where $\overline{f}$ is micro-unit strength variable, m and f_0 are distribution parameters.

The loading damage variable is defined as the ratio of the quantity of failure unit N_f to that of total unit N:

$$D_l = \frac{N_f}{N} \tag{3}$$

The quantity of failure unit in interval $[0,\overline{f}\,]$ can be computed as

$$N_f(\overline{f}\,) = \int_0^{\overline{f}} NP(y)dy = N\left\{1 - \exp\left[-\left(\frac{\overline{f}}{f_0}\right)^m\right]\right\} \tag{4}$$

Then we can obtain the loading damage variable by substituting Eq. (4) into Eq. (3):

$$D_l = 1 - \exp\left[-\left(\frac{\overline{f}}{f_0}\right)^m\right] \tag{5}$$

The micro-unit strength is assumed to satisfy the maximum-tensile-strain yield criterion which can be written as

$$\overline{f} = f(\varepsilon) = \varepsilon \tag{6}$$

where ε is the maximum strain.

Substituting Eq. (6) into Eq. (5), the loading damage variable is

$$D_l = 1 - \exp\left[-\left(\frac{\varepsilon}{f_0}\right)^m\right] \tag{7}$$

By generalized Hooke's law, the stress-strain relationship of rock under loading can be written as

$$\overline{\sigma}_{ij} = 2G\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk} \tag{8}$$

where $G = \frac{E_0}{2(1+\nu)}$, $\lambda = \frac{E_0\nu}{(1+\nu)(1-2\nu)}$; G and λ are shear modulus and lame constant of rock, respectively; E_0 and ν are static elastic modulus and Poisson's ratio of fresh rock, respectively.

Generally the stress used in laboratory experiment is nominal stress, then substituting Eq. (8) into Eq. (1) and eliminating damage variable we can derive the effective stress expressed by nominal stress:

$$\overline{\sigma}_i = \frac{\sigma_i E_0 \varepsilon_1}{\sigma_1 - \nu(\sigma_2 + \sigma_3)} \tag{9}$$

Substituting Eqs. (1) and (7) into Eq. (9), the constitutive equation of rock under loading is

$$\sigma_i = \nu(\sigma_2 + \sigma_3) + E_0\varepsilon_1 \exp\left[-\left(\frac{\varepsilon_i}{f_0}\right)^m\right] \tag{10}$$

The process of damage under freeze-thaw and loading is considered to be loading damage after freeze-thaw. The freeze-thaw damage variable of rock can be expressed by static elastic modulus as below:

$$D_t = 1 - \frac{E_n}{E_0} \tag{11}$$

where E_0 is the static elastic modulus of fresh rock; E_n is the static elastic modulus of freeze-thaw damage rock after n cycles; D_t is freeze-thaw damage variable.

Considering freeze-thaw damage, the statistical damage constitutive equation of rock under freeze-thaw and loading can be obtained by replacing E_0 with E_n:

$$\sigma_i = \nu(\sigma_2 + \sigma_3) + E_n\varepsilon_1 \exp\left[-\left(\frac{\varepsilon_i}{f_0}\right)^m\right] \tag{12}$$

It's shown that there are three unknown parameters E_n, m and f_0 needing to be determined.

2.2. Revolution rule of static elastic modulus under freeze-thaw

The static elastic modulus of rock decreases with the increasing of freeze-thaw cycles (Xu and Liu, 2005; Zhang and Yang, 2013; Li et al., 2014; Fang et al., 2014; Wu et al., 2014). However, generally what we can get are the measured values of rock parameters after several specific freeze-thaw cycles. So predicting the value of static elastic modulus after any cycle is crucial for the present model. Dynamic parameters are easier to derive by experiment than static parameters in the process of freeze-thaw cycles, such as P-wave velocity. Thus, if we can deduce the relation between P-wave velocity and static elastic modulus, the static elastic modulus can be easily predicted by P-wave velocity (Liu et al., 2015).

Palmström and Singh (2001) raised that the ratio of dynamic elastic modulus to static elastic modulus in fresh rock and that in rock mass would keep unchanged:

$$\frac{E_r'}{E_m'} = \frac{E_r}{E_m} \tag{13}$$

where E_r' and E_r are dynamic and static elastic modulus of fresh rock, respectively; E_m' and E_m are dynamic and static elastic modulus of rock mass, respectively.

Fresh rock is undamaged material while there is a large amount of cracks and pores in damaged rock mass. Similarly, the fresh rock will be damaged after freeze-thaw cycle. Let's assume that the ratio of dynamic elastic modulus to static elastic modulus in fresh rock and that in damaged rock after freeze-thaw also keeps unchanged. Thus, as well as Eq. (13) we can derive

$$\frac{E_d}{E_0} = \frac{E_{nd}}{E_n} \tag{14}$$