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Stochastic analysis of uncertainty mechanical characteristics for surrounding rock and lining in cold region tunnels

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ABSTRACT

The mechanical characteristics of tunnels in cold regions are uncertain because of the randomness of temperature field and mechanical parameters. Conventional forecast analysis is always deterministic, rather than taking stochastic temperature and parameters into account. This paper aims to investigate the stochastic mechanical characteristics of tunnel in cold regions on the basis of a stochastic analysis model and stochastic finite element method. A series of computer codes is compiled by Matrix Laboratory (MATLAB) software, and the stochastic mechanical characteristics for a tunnel in a cold region are obtained and analyzed by Neumann stochastic finite element method (NSFEM). The results provide a new method to predict the uncertain mechanical characteristics of tunnel in cold regions, and it shows that the stochastic temperature has an effect on stochastic stress for surrounding rock, and the major principle stress and minor principle stress are changeable in different times. The frost heave of surrounding rock leads to the increase of mean stress and mean displacement for lining while the thawing process leads to the decline of mean stress and mean displacement for lining. With the passage of time, the stress standard deviation and displacement standard deviation for lining show an upward trend, which imply that the results of conventional deterministic analysis for mechanical characteristic of tunnel lining may be farther from the true value. These results can improve our understanding of the uncertainty mechanical characteristics for tunnel and provide a theoretical basis for engineering reliability analysis and design in cold regions.

1. Introduction

With the dramatic development of economy in northwestern China, a great deal of infrastructures has been built in the past decades in cold regions. High-speed transportation system, such as high-speed way and railway, is one of the most important projects by the central government. Many tunnels have been inevitably constructed due to many mountains and plateaus. In the constructed tunnel engineering through cold regions, the rocks surrounding the tunnels are affected by the seasonal variation of the air temperatures, which will accelerate the changes of the thermal state (Lai et al., 2005; Zhang et al., 2002, 2004, 2006). For different conditions, frozen surrounding rock or thawed surrounding rock, the thermodynamic property will have a big difference. Therefore, the temperature change will lead to a series of mechanical behavior variations of the surrounding rock, which have an adversely impact on the mechanical state of the tunnels, and seriously endanger the safety (Yang et al., 2006; Gao et al., 2012). Thus far, many literatures have been trying to solve the problems of mechanical characteristics and disaster forecasts for the tunnels in cold regions (Lai et al., 1998, 2000; Lu et al., 2011; Feng et al., 2014). It is obvious that the numerical methods, such as finite element method, have the advantages of low cost and short research turnaround. With the development of the computer, numerical methods become increasingly popular, providing useful information for cold regions tunnels. However, all of the researches of mechanical characteristic for tunnels in cold regions are developed under the assumption that the mechanical parameters and thermal regime are deterministic.

In fact, the property parameters of rock and soil are variable because of the complex geological processes (Dasaka and Zhang, 2012; Bong et al., 2014; Chen et al., 2015; Liu et al., 2017). Especially for cold regions, the structure of frozen rock and soil vary with the random distribution of internal defects. Therefore, its mechanics properties exhibit randomness and uncertainty, and the stress-strain relationship, especially of warm frozen soil and warm ice-rich frozen soil, cannot be described well deterministically (Lai et al., 2008, 2012). Furthermore, some scholars paid their attention on the stochastic thermal analysis in permafrost regions (Liu et al., 2014; Wang et al., 2015), and the random temperature fields of tunnel in a cold region are obtained by Neumann

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stochastic finite element method (Wang et al., 2016). For frozen surrounding rock, it is obvious that the randomness of rock temperature will lead to the randomness of mechanical parameters because they are closely related. Therefore, it is extremely significant to consider the stochastic aspects of the temperature and parameters when the stability analysis of mechanical characteristics for tunnel in cold regions is to be conducted, especially for the variability of maximum stress and maximum displacement.

This paper aimed to forecast the uncertainty mechanical characteristics of tunnel in cold regions. Based on the earlier studies of stochastic analysis for the uncertain temperature field of tunnel (Wang et al., 2016), considering the randomness of mechanical parameters and thermal regime, a stochastic analysis method of uncertain mechanical characteristics for tunnel in cold regions is developed. The stochastic stress and deformation are obtained by NSFEM. Through distributions of mean and standard deviation, the rules and process of stochastic mechanical characteristics are analyzed and predicted in detail, especially for maximum stress and maximum displacement. From this study, some useful conclusions will be drawn, which can provide theoretical basis and reference for design, maintenance and research on the tunnel engineering in cold regions.

2. Governing equations and finite element formulae

In rock mechanics, the compression is often defined as positive. Based on the principle of virtual work (Davis and Selvadurai, 2002), the equilibrium equation for deterministic finite element analysis can be written as

$$\int_V \delta\{\varepsilon\}^{\mathrm{T}}\{\sigma\}_t dV = \int_V \delta\{u\}^{\mathrm{T}}\{f\}_t dV + \int_S \delta\{u\}^{\mathrm{T}}\{\overline{f}\}_t dS \tag{1}$$

where V is the volume; S is the surface; subscript t is the load step; $\{\sigma\}_t$ is the stress vector; $\{f\}_t$ is the body force vector; $\{\overline{f}\}_t$ is the surface force vector; $\delta\{\varepsilon\}$ is the virtual strain vector; $\delta\{u\}$ is the virtual displacement vector.

The displacement of internal location for each finite element mesh is expressed as follows:

$$\{u\} = [N]\{\delta\} \tag{2}$$

where $\{\delta\}$ is the node displacement vector; $[N]$ is the interpolation function matrix.

Based on the assumption that strain is small and compressive strain is assumed positive, the strain-displacement relation of rock can be defined as:

$$\{\varepsilon\} = [B]\{\delta\} \tag{3}$$

where $\{\varepsilon\}$ is the strain vector; $[B]$ is the geometric function matrix.

Substituting Eqs. (2), (3) into (1), the deterministic finite element formulae can be obtained, namely:

$$\int_V [B]^{\mathrm{T}}\{\sigma\}_t dV = \int_V [N]^{\mathrm{T}}\{f\}_t dV + \int_S [N]^{\mathrm{T}}\{\overline{f}\}_t dS \tag{4}$$

Then, the incremental finite element formulae can be written as:

$$\int_V [B]^{\mathrm{T}}\{\Delta\sigma\} dV = \int_V [N]^{\mathrm{T}}\{\Delta f\} dV + \int_S [N]^{\mathrm{T}}\{\Delta\overline{f}\} dS \tag{5}$$

When the initial conditions take body force vector into account, Eq. (5) can be simplified as:

$$\int_V [B]^{\mathrm{T}}\{\Delta\sigma\} dV = \int_S [N]^{\mathrm{T}}\{\Delta\overline{f}\} dS \tag{6}$$

In this paper, the Mohr-Coulomb model is adopted, and the yield function can be written as:

$$f = p\sin\varphi + \frac{q}{6}[(\cos\theta - \sqrt{3}\sin\theta)\sin\varphi - (3\cos\theta + \sqrt{3}\sin\theta)] + c\cos\varphi \tag{7}$$

where f is yield surface; p is the spherical stress; q is the deviatoric stress; φ is the angle of internal friction; θ is the Lode angle; and c is the cohesion.

Based on the classical elastic-plastic theory (Zheng et al., 2002), the stress-strain relationship is expressed as follows:

$$\{d\sigma\} = [D]^{ep}\{d\varepsilon\} \tag{8}$$

$$[D]^{ep} = [D]^e - \frac{[D]^e\left\{\frac{\partial g}{\partial\sigma}\right\}\left\{\frac{\partial f}{\partial\sigma}\right\}^{\mathrm{T}}[D]^e}{A + \left\{\frac{\partial f}{\partial\sigma}\right\}^{\mathrm{T}}[D]^e\left\{\frac{\partial g}{\partial\sigma}\right\}} = [D]^e - [D]^p \tag{9}$$

where $[D]^{ep}$ is the elastic-plastic constitutive matrix; $[D]^e$ is the elastic constitutive matrix; $[D]^p$ is the plastic constitutive matrix. g is the plastic potential function, and $g = f$.

Substituting Eqs. (7), (9) into (8), the whole process of elastic-plastic stress-strain relationship can be calculated. According to the algorithm of variable stiffness in finite element method (Xie and He, 1981; Meng, 1985), the finite elements of computational domain can be divided into three kinds of element, namely elastic elements, plastic elements and transitional elements. Therefore, there are three kinds of element domain. The computational domain can be written as:

$$V = V^e + V^p + V^g \tag{10}$$

According to Eqs. (6), (8) and (10), the following finite element formulae are obtained.

$$[K]\{\Delta\delta\} = \{\Delta R\} \tag{11}$$

$$[K] = \int_{V^e} [B]^{\mathrm{T}}[D]^e[B]dV + \int_{V^p} [B]^{\mathrm{T}}[D]^p[B]dV + \int_{V^g} [B]^{\mathrm{T}}[D]^g[B]dV \tag{12}$$

where $[K]$ is the stiffness matrix; $\{\Delta R\}$ is the increment of equivalent nodal forces vector. $[B]$ is the element strain matrix; $[D]^g$ is the transitional matrix.

Based on the algorithm of variable stiffness (Meng, 1985), the transitional matrix can be written as:

$$[D]^g = \overline{m}[D]^e + (1 - \overline{m})[D]^p \tag{13}$$

$$\overline{m} = \frac{\Delta\overline{\sigma}_A}{\Delta\overline{\sigma}_B} \tag{14}$$

where $\overline{m}$ is the weighting coefficient; $\Delta\overline{\sigma}_A$ is the increment of equivalent stress when the finite element reaches its yield. $\Delta\overline{\sigma}_B$ is the increment of equivalent stress caused by load step.

3. Analysis method for uncertainty mechanical characteristics

Based on the random field theory (Vanmarcke, 1977, 1983), we consider the spatial variability of mechanics properties and model the elastic modulus, cohesion, angle of internal friction and Poisson ratio as 2D random fields. When the 2D random field is divided by triangular elements (Wang et al., 2014), the covariance of two local average elements is

$$Cov(X_e, X_{e'}) = \sigma^2 \sum_{K=1}^{M}\sum_{R=1}^{M} \omega^{(K)}\omega'^{(R)} g(N_i^{(K)}, N_j^{(K)}, N_k^{(K)}, N_i'^{(R)}, N_j'^{(R)}, N_k'^{(R)}) \tag{15}$$

where N_i, N_j and N_k are the shape functions of three nodes of element e, respectively; N_i', N_j' and N_k' are the shape functions of three nodes of element e', respectively; M is the number of basis points; $\omega^{(K)}$ is the weighted coefficient of e and $\omega'^{(R)}$ is the weighted coefficient of e'.

Eq. (15) is the calculating formula of the covariance for two local average elements. If the standard correlation function is known, we can obtain the result of covariance matrix. In order to guarantee the calculation accuracy, we assume that $M = 7$, and Table 1 is the calculating parameter of Eq. (15). Because the covariance matrices obtained from Eq. (15) are full-rank matrices, calculating the covariance matrices is inefficient. Therefore, a set of uncorrelated random variables is