Earth and Planetary Science Letters 498 (2018) 397–407

Contents lists available at ScienceDirect

Earth and Planetary Science Letters

www.elsevier.com/locate/epsl

Pressure variations among rheologically heterogeneous elements in Earth's lithosphere: A micromechanics investigation

Dazhi Jiang [a,b,*], Ankit Bhandari [a]

[a] Department of Earth Sciences, Western University, London, Ontario N6A 5B7, Canada
[b] Department of Geology, Northwest University, Xi'an, China

ARTICLE INFO

Article history:
Received 22 February 2018
Received in revised form 1 July 2018
Accepted 7 July 2018
Available online 23 July 2018
Editor: A. Yin

Keywords:
tectonic pressure
micromechanics
Eshelby formalism
rheological anisotropy
MOPLA

ABSTRACT

The total pressure a rock element is subjected to in Earth's lithosphere undergoing deformation may deviate from the lithostatic pressure. Despite many decades of research, the significance of this pressure deviation is still debated. Here, we apply the micromechanics approach based on the generalized Eshelby inclusion solutions for anisotropic power-law viscous materials to investigate the pressure deviation in rheologically heterogeneous rocks. We regard a rheologically distinct element (RDE) as a microscale heterogeneous inclusion which is embedded in and interacts with the heterogeneous macroscale medium. The latter is represented by a homogeneous-equivalent medium (HEM) with its effective rheology obtained self-consistently from the rheological properties of all constituent elements making up the ambient macroscale material embedding the RDE. Partitioning equations from the generalized Eshelby solutions and developed numerical implementations allow the pressure deviations inside and around the RDE to be calculated with quasi-analytical accuracy. We prove formally for limiting cases and demonstrate numerically that the maximum pressure deviation in and around any RDE is on the same order as the deviatoric stresses in the ambient medium or in the element for general power-law viscous materials, isotropic or anisotropic. The pressure deviation fields related to a RDE in an anisotropic HEM are lower than the pressure deviation fields related to the same RDE in an isotropic HEM. Pressure deviations due to an initial pressure anomaly are insignificant considering the viscoelastic interaction between an RDE and the HEM. Our results suggest that pressure deviations in Earth's lithosphere are insignificant for metamorphic processes if the differential stress in Earth's lithosphere is on a few hundred MPa level. Higher pressure deviations require correspondingly greater differential stresses in the lithosphere. They may be generated by short-term strong elastic interactions.

1. Introduction

Pressure is an important variable in nearly all geological processes. Pressure estimates from mineral assemblages using various geothermobarometers (e.g., Spear, 1993) are routinely used as a proxy for the depth at which the assemblages were formed. This in turn is used to build geodynamic models for the geological process. However, it has been realized for many decades (Gerya, 2015; Mancktelow, 2008 and references therein) that tectonic deformation may cause the local pressure in a volume of rock to differ from the lithostatic pressure. This pressure difference has been called the 'tectonic overpressure' or 'underpressure', depending on whether the difference is positive or negative, in the literature. In this paper, we shall use *tectonic pressure deviation* or simply *pressure deviation* to refer to the pressure difference caused by deformation because only pressure deviations from a mean value are relevant in petrological records whereas the lithostatic pressure is unknown *a priori*. As established thermodynamic principles dictate that mineral assemblages are related to the *total* pressure, to convert geobarometric data to depth, it is critical to know how significant the error range may be due to the presence of possible tectonic pressure deviation. This includes the level of the tectonic pressure deviation, its variation in space from one geological unit to another at a given scale and across different scales, and the timespan an elevated pressure deviation (if generated) may be sustained. Although these problems have been tackled for over two decades through analytical (e.g., Mancktelow, 2008, 1995, 1993; Schmalholz et al., 2014b) and numerical modeling approaches (e.g., Burov et al., 2014; Li et al., 2010; Reuber et al., 2016), the results of these works are still controversial and inconsistent.

* Corresponding author at: Department of Earth Sciences, Western University, 1151 Richmond Street, London, Ontario N6A 5B7, Canada.
E-mail address: djiang3@uwo.ca (D. Jiang).

https://doi.org/10.1016/j.epsl.2018.07.010
0012-821X/

These works have shown quite clearly that the magnitude and variation of the tectonic pressure deviation are rather sensitive to the model geometry, boundary conditions, rheologies and rheological parameters assigned to the model elements. Simple models based on Jaeger (1969, pp. 140–149) that have analytical solutions have been applied to natural transpressional zone deformation (Robin and Cruden, 1994) and extrusion or convergent channel systems (Mancktelow, 2008, 1995, 1993; Raimbourg and Kimura, 2008) and GPa-level pressure deviations have been predicted in the deforming zone (Mancktelow, 2008). However, these treatments regard the transpressional zone or the convergent channel as a tabular 'deforming zone' sandwiched between rigid or nearly rigid walls (country rocks) moving at a constant velocity relative to each other. The interface between the country rocks and the deforming zone is assumed to be fixed to material and is non-slippery. These assumptions lead to unrealistically strong mechanical interactions between the deforming zone and the country rocks and are, as we argue in this paper, responsible for the predicted GPa-level pressure deviations. Many 2D numerical models limited to elements of isotropic rheologies for large scale collisional tectonic scenarios (e.g., Burg and Gerya, 2005; Burov et al., 2014; Li et al., 2010; Schmalholz et al., 2014a) have considered more reasonable boundary conditions. Differing results have been obtained with some getting 'significant' (>20% of the lithostatic value) (e.g., Burg and Gerya, 2005; Schmalholz et al., 2014a) and others predicting 'insignificant' (<20% of the lithostatic value) pressure deviation (e.g., Burov et al., 2014; Li et al., 2010). As each numerical modeling investigation uses a distinct computational procedure, it is not possible (e.g., Post and Votta, 2005) to identify how the inconsistency has arisen. It seems that the model results depend strongly on the loading condition and the choice of rheology and rheological parameters. The pressure deviation problem has also been analyzed using 2D inclusion solutions for isotropic Newtonian materials (Mancktelow, 2008; Moulas et al., 2014; Schmid and Podladchikov, 2003). Mancktelow (2008) concluded that the pressure deviations related to strong inclusions are of order 1–2 times the maximum shear stress the strong material can support. How the conclusion may be affected by power-law rheology, rheological anisotropy, and 3D inclusion deformation, which are all more relevant to natural deformation, is unknown.

In this paper, we apply a full mechanical approach – the generalized Eshelby's inclusion solutions for power-law viscous materials (Jiang, 2016, 2014) to investigate the pressure deviation in natural deformation. This approach avoids unrealistic mechanical interactions caused by assumptions on rheological behaviors and boundary conditions and addresses the 3D deformation and anisotropic viscosity. Specifically, we regard the ductily-deforming rock masses undergoing metamorphism as a composite material made of rheologically distinct elements (RDEs). A RDE can represent any rheological heterogeneity, such as a distinct lithological unit or structural element like a ductile shear zone. We consider the long-term deformation with characteristic time scales on Ma, much greater than the viscous relaxation time (discussed later) so that elastic behaviors can be ignored. All RDEs and the composite material as a whole are assumed to be power-law viscous. Supposing that a large representative volume of this 'lithosphere material' is subjected to a given macroscale deformation, we are concerned with how the pressure varies from one RDE to another and deviates from the bulk ambient pressure. To obtain the pressure deviation inside and around any RDE from the ambient pressure, we regard the RDE as an Eshelby inclusion embedded in the medium. The macroscale medium surrounding the RDE is rheologically heterogeneous, but in micromechanics it is represented by a hypothetical homogeneous-equivalent medium (HEM) whose rheology is obtained self-consistently from the rheological properties

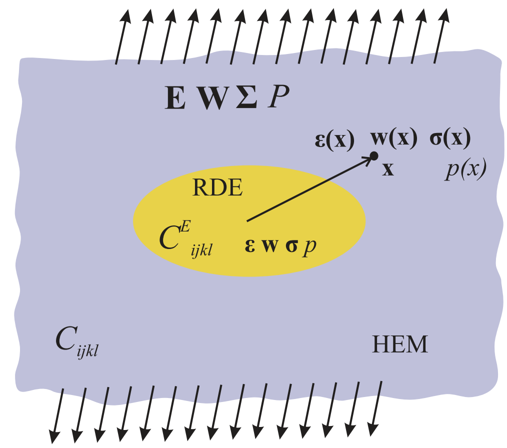

Fig. 1. The Eshelby inclusion problem of an ellipsoidal RDE in a HEM and symbols used in partitioning Eqs. (1) and (2). $\mathbf{C}^E_{ijkl}$ and $\mathbf{C}_{ijkl}$ are the viscous stiffness of the RDE and HEM respectively. The far-field mechanical state is defined by upper case symbols $\mathbf{\Sigma}$, $\mathbf{E}$, $\mathbf{W}$, and P, denoting respectively the deviatoric stress, strain rate, vorticity, and pressure. The constant mechanical fields inside the ellipsoid (the interior fields) are denoted by corresponding lower case symbols, $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$, $\mathbf{w}$, and p, and the difference fields are represented by $\widetilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma} - \mathbf{\Sigma}$, $\widetilde{\mathbf{w}} = \mathbf{w} - \mathbf{W}$, and $\widetilde{p} = p - P$. The mechanical fields around the inclusion (exterior fields) vary with the position vector $\mathbf{x}$ and are expressed by $p(\mathbf{x})$ etc. and $\widetilde{p}(\mathbf{x}) = p(\mathbf{x}) - P$.

of all constituent elements contained in a representative volume element (RVE) that embeds the RDE. Generalized Eshelby inclusion solutions for viscous power-law materials (Jiang, 2016, 2014, 2013) relate formally the local mechanical fields (including pressure) in and around the RDE to the macroscale mechanical fields. Pressure deviations inside and around the RDE can be computed with quasi-analytical accuracy using the partitioning equations from the Eshelby inclusion solutions. Because the formalism considers 3D deformation, non-linear viscous rheology, and rheological anisotropy, we can systematically investigate the pressure deviation as functions of these variables. As a thorough treatment of the generalized Eshelby solutions is given in Jiang (2016, 2014), only the partitioning equations used in this paper are presented in the following section.

2. Partitioning equations

The classical Eshelby inclusion/inhomogeneity problem is illustrated with Fig. 1. Initially, Eshelby (1959, 1957) solved for the elastic field inside and outside an isolated ellipsoidal domain which he called an "inclusion", if it has identical elastic properties as the surrounding medium, and an "inhomogeneity" if the domain has distinct elastic properties. For simplicity, we use "inclusion" to refer to any heterogeneous element in a composite material in this paper following Jiang (2016, 2014) unless a distinction must be made for clarity. Eshelby's elegant point-force method and equivalent-inclusion approach have been extended to general anisotropic linear elastic materials as reviewed and summarized in Mura (1987) and general anisotropic linear viscous materials (see reviews of Jiang, 2016, 2014). The partitioning equations for the mechanical fields inside the inclusion are given in the following set (Jiang, 2016, Eqs. (12) there):

$$\widetilde{\boldsymbol{\varepsilon}} = \left[\mathbf{J}^{\mathrm{d}} - \mathbf{S}^{-1}\right]^{-1} : \mathbf{C}^{-1} : \widetilde{\boldsymbol{\sigma}} \tag{1a}$$

$$\widetilde{\mathbf{w}} = \mathbf{\Pi} : \mathbf{S}^{-1} : \widetilde{\boldsymbol{\varepsilon}} \tag{1b}$$

$$\widetilde{p} = \mathbf{\Lambda} : \mathbf{C} : \mathbf{S}^{-1} : \widetilde{\boldsymbol{\varepsilon}} \tag{1c}$$

In Eqs. (1), the sign ":" stands for double-index contraction of two tensors. Lowercase and uppercase symbols (Fig. 1) stand for