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The global magnitude–frequency relationship for large explosive volcanic eruptions

Jonathan Rougier [a,*], R. Stephen J. Sparks [b], Katharine V. Cashman [b], Sarah K. Brown [b]

[a] *School of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, UK*
[b] *School of Earth Sciences, University of Bristol, Wills Memorial Building, Queens Road, Bristol BS8 1RJ, UK*

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ABSTRACT

For volcanoes, as for other natural hazards, the frequency of large events diminishes with their magnitude, as captured by the magnitude–frequency relationship. Assessing this relationship is valuable both for the insights it provides about volcanism, and for the practical challenge of risk management. We derive a global magnitude–frequency relationship for explosive volcanic eruptions of at least 300 Mt of erupted mass (or M4.5). Our approach is essentially empirical, based on the eruptions recorded in the LaMEVE database. It differs from previous approaches mainly in our conservative treatment of magnitude-rounding and under-recording. Our estimate for the return period of 'super-eruptions' (1000 Gt, or M8) is 17 ka (95% CI: 5.2 ka, 48 ka), which is substantially shorter than previous estimates, indicating that volcanoes pose a larger risk to human civilisation than previously thought.

1. Introduction

There are both fundamental science reasons and practical reasons for establishing a global relationship between magnitude and frequency for explosive volcanic eruptions. The magnitude–frequency relationship constrains rates of volcanism, provides potential insights into the underlying tectonic and igneous processes that control volcanism and establish the conditions for explosive eruptions, and provides critical information to forecast future eruptions and assess attendant volcanic hazards, including the effects on climate of large explosive eruptions.

More broadly, interest in extreme geohazard events and their consequences is increasing following a series of high-profile earthquakes, tropical cyclones and tsunamis that have had substantial regional impacts (e.g., Plag et al., 2015). From this perspective, the frequency of very large explosive eruptions is of particular importance due to the potential for such eruptions to have not only regional but also global environmental and societal effects. Although the magnitude–frequency relationship for large-magnitude eruptions has been well-studied (Pyle, 1995; Siebert et al., 2010; Deligne et al., 2010; Sheldrake and Caricchi, 2017), some uncertainty remains, while the relationship for the largest-magnitude explosive eruptions is not well known (although see Mason et al., 2004).

The challenge for estimating the magnitude–frequency relationship is that large explosive eruptions are rare. Records of the largest eruptions are extracted from proxies in geological archives. Naturally, such proxies are hard to interpret, and the resulting values for dating and magnitude have substantial uncertainties and may be systematically biased. The frequency of eruptions in a modern database is also misleading, because the probability of an historical eruption leaving a trace that survives to be found and included in the database depends on the time, location, and magnitude of the eruption. Thus, incautious use of recorded large eruptions can lead to an inaccurate estimate of the magnitude–frequency relationship. Our approach in this paper is conservative with respect to mis-recording, and all of our point estimates are accompanied by 95% confidence or credible intervals.

The plan of the paper is as follows. Section 2 describes the scale for magnitude, and two complementary ways to present the magnitude–frequency relationship: the exceedance probability curve and the return period curve. Section 3 describes the database and the records it contains, highlighting two sources of inaccuracy. Section 4 describes our statistical model, and uses it to estimate a semi-parametric approximation of the exceedance probability curve. Section 5 introduces a parametric model better able to accommodate the limitations in the records. Section 6 presents our preferred estimate of the exceedance probability curve, based

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[*] Corresponding author.
E-mail addresses: j.c.rougier@bristol.ac.uk (J. Rougier), steve.sparks@bristol.ac.uk (R.S.J. Sparks), kathy.cashman@bristol.ac.uk (K.V. Cashman), sarah.k.brown@bristol.ac.uk (S.K. Brown).

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on the parametric model, and compares our estimates of the return period with others in the literature. Section 7 concludes with a summary and a brief discussion of the implications of our estimate.

2. The magnitude–frequency relationship

The magnitude scale is

$$M = \log_{10}(\text{erupted mass in kg}) - 7, \tag{1}$$

as defined by Pyle (2000) and Mason et al. (2004). We prefer this scale to the widely used Volcanic Explosivity Index (VEI, see Newhall and Self, 1982) because VEI is ordinal and so cannot be represented by a continuous function to describe magnitude and frequency. Further, VEI is assigned to an eruption based on multiple criteria, including eruption column height, which cannot be directly related to magnitude, so VEI is not consistently a measure of magnitude. However, the legacy of VEI creates difficulties in interpreting records of previous eruptions, as discussed in section 3.

The global magnitude–frequency relationship for large explosive eruptions can be represented in two complementary ways. First, in terms of the 'exceedance probability' curve, here denoted $\bar{P}$. The value $\bar{P}(m)$ is the probability of at least one eruption of at least magnitude m happening somewhere in the world in the next year. The largest recorded eruption since 100 ky is Toba (Indonesia), dated 73 ky, recorded at $M = 9.1$ (Costa et al., 2014). The value $\bar{P}(9.1)$ is the probability of another Toba (or worse) happening in the next year. In this paper we use 'My' and 'ky' to denote a point in time in years BP, and 'Ma' and 'ka' to denote a duration.

Second, the magnitude–frequency relationship can be represented in terms of the 'return period' curve, denoted R. The value $R(m)$ is the mathematical expectation of the time to wait until an eruption with magnitude of at least m. Thus $R(9.1)$ is the expected time to wait, in years, until an eruption which is at least as large as Toba.

Both the exceedance probability curve and the return period curve can be derived within a stochastic process model for eruption times and magnitudes. In our marked Poisson process model they are complementary, because $R(m) \approx 1/\bar{P}(m)$ if $\bar{P}(m)$ is small (see section 6). However, the two labels '$\bar{P}(m) = 0.001$' and '$R(m) = 1000$ years' will often be interpreted differently by non-experts. The latter seems more user-friendly, but can give a very misleading impression, particularly in a changing environment (although this is more relevant to flooding than to volcanoes).

There is another reason for preferring exceedance probabilities over return periods, which is both technical and practical. The time to wait until an eruption is an unbounded quantity, and consequently the value of its expectation is susceptible to very large values occurring with small probabilities; in fact, the expectation may be infinite, particularly when integrating out the parameters in a Bayesian approach. This is a general problem with expectations: they can provide poor summary values for unbounded quantities. Therefore, we prefer to represent the magnitude–frequency relationship as the exceedance probability curve. Where return periods are required, we adopt the convention of using the reciprocal of the exceedance probability, providing that this probability is small.

3. The volcanic record

The Large Magnitude Explosive Volcanic Eruptions database (LaMEVE) provides a global compilation of data on magnitudes and ages during the Quaternary (Crosweller et al., 2012; Brown et al., 2014). LaMEVE has been developed to complement the Volcanoes of the World (VOTW) database of the Smithsonian Institution for the Holocene and is based on literature for pre-Holocene entries. This analysis is based on version 3.1 of the database, released in Oct. 2015. However, in the light of our preliminary results we initiated a revision of all records of eruptions since 100 ky with $M \geq 7$, and some uncertain records at lower magnitudes. The results will be incorporated into the next version of LaMEVE, but in the meantime our dataset is available as a spreadsheet in the supplementary information to this paper.

This paper focuses on records in LaMEVE that are dated to have occurred since 100 ky, 1379 eruptions in total. This section considers the difficulties in interpreting these records. One difficulty which we need not consider, except in passing, is the challenge of dating an eruption from its trace in the geological record. This is because we sidestep dating uncertainty by using a statistical model which is time-invariant, at the global scale. This 'stationarity' assumption is discussed in more detail in section 4.

3.1. Magnitude accuracy

Pyle (2016) summarises the methods for assessing magnitude from geological data, and the many sources of error, and thus of uncertainty. He does not provide uncertainty estimates. However, an assessment of volume estimates from isopach maps of tephra fall deposits with at least 20 data thickness points indicates uncertainties typically exceeding $M \pm 0.3$ (Engwell et al., 2015).

Measurement errors are fairly unsystematic, being a source more of noise than of bias. However, inspection of the frequencies of recorded magnitudes reveals a systematic error and thus a potentially large source of bias. The lefthand panel of Fig. 1 shows that recorded frequencies pile-up on the integer magnitude values, which must be an artefact; see also Brown et al. (2014).

By going back through the database and the supporting papers, we identified one source of rounding. A subset of the records are eruptions with a recorded VEI of v (an integer) but without a reported magnitude, and these were coded as $M = v.0$. However, a VEI value of v corresponds to a magnitude of $v.0$ to $v.9$. There were 163 such eruptions in records dated since 100 ky. This is 'rounding down', which shifts the exceedance probability downwards, understating the exceedance probability of large explosive eruptions, and overstating the length of the return period for large explosive eruptions.

Fig. 1 also shows the frequencies of recorded magnitudes after removing the subset identified above. The frequencies still pile-up on the integer magnitude values, indicating that there is another source of rounding. The righthand panel of Fig. 1 shows that widening the bins from width 0.1 to width 0.5 does not remove the piling up. We suspect that this source is rounding towards the nearest integer. We speculate—and it is no more than that—that a volcanologist who assesses a magnitude that is close to an integer may well round to the integer, in the light of her own assessment of uncertainty, in order not to give a spurious impression of accuracy. However, as a reviewer notes, there is an issue about whether the volcanologist assesses volume and then rounds, and then the rounded value is converted to mass using a standard density such as 2500 kg/m^3, or whether the volcanologist assesses mass directly and rounds that. In due course a better operational understanding of rounding might change our results. We return to this topic in the discussion of Table 2 in section 6.

In order to make progress, we will group the recorded magnitudes into integer-width bins centred at the integers, reflecting our view, supported by Fig. 1, that rounding to the nearest integer is the dominant source of piling-up on the integer magnitude values. Any aggregation into bins will reduce the effect of rounding, even if it does not remove it completely. We will exclude recorded magnitudes below $M = 4.5$ for which there is no integer-width bin, because the LaMEVE database is for $M \geq 4$. Further screening for under-recording, described immediately below, removes all but one of the records in the rounding-down subset identified above,

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