Geoscience Frontiers xxx (2017) 1–6

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Geoscience Frontiers

journal homepage: www.elsevier.com/locate/gsf

Research paper

Variation of singularity of earthquake-size distribution with respect to tectonic regime

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ARTICLE INFO

Article history:
Received 20 February 2017
Received in revised form
6 April 2017
Accepted 14 April 2017
Available online xxx
Handling Editor: Masaki Yoshida

Keywords:
Fractals
Singularity
Power-law distribution
Plate tectonics
Mathematical model
Spatial distribution

ABSTRACT

Frequency–size relation of earthquakes in a region can be approximated by the Gutenberg-Richter law (GR). This power-law model involves two parameters: a-value measuring seismic activity or earthquake productivity, and b-value describing the relation between frequencies of small and large earthquakes. The spatial and temporal variations of these two parameters, especially the b-value, have been substantially investigated. For example, it has been shown that b-value depends inversely on differential stress. The b-value has also been utilized as earthquake precursor in large earthquake prediction. However, the physical meaning and properties of b-value including its value range still remain as an open fundamental question. We explore the property of b-value from frequency–size GR model in a new form which relates average energy release and probability of large earthquakes. Based on this new form of GR relation the b-value can be related to the singularity index (1–2/3b) of fractal energy – probability power-law model. This model as applied to the global database of earthquakes with size $M \geq 5$ from 1964 to 2015 indicates a systematic increase of singularity from earthquakes occurring on mid-ocean ridges, to those in subduction zones and in collision zones.

1. Introduction

Earthquakes caused by self-organized criticality (SOC) like avalanches often generate anomalous energy release within narrow spatial-temporal intervals. Patterns of spatial, temporal and magnitude distribution of earthquakes have been the focus of many studies and been used as the main attributes involved in models for prediction. It has been demonstrated that the energy release by earthquakes follows power-law relations with accumulative time, scale of space and frequency of earthquakes. For example, the relationship between the accumulative number of earthquakes $N(>M)$ with magnitude greater than M can be expressed as the Gutenberg-Richter relation (Gutenberg and Richter, 1944)

$$\log N(>M) = \mathrm{a} - \mathrm{b}M \tag{1}$$

where M is the magnitude of an earthquake, $N(>M)$ is accumulative number of earthquakes with magnitude greater than threshold M, while a and b are two parameters determining the power-law function. According to the energy – magnitude relation (Gutenberg and Richter, 1956), $\log E \propto 1.5M$, where the symbol $\propto$ stands for proportional to, the Gutenberg-Richter equation indicates that earthquake magnitude satisfies a logarithmic transformation of seismic moment E (energy) so that the GR relation can be rewritten as

$$N(>E) = cE^{-\frac{2}{3}\mathrm{b}} \tag{2}$$

Eqs. (1) and (2) have been commonly used in describing the relation between frequency and size distribution of earthquakes. Many researchers have investigated the temporal and spatial variation of the parameter b with respect to other physical properties (Smith, 1981; Turcotte, 1992; Frohlich and Davis, 1993; Okal and Romanowicz, 1994; Kagan, 1997; Schorlemmer et al., 2005). For example, the temporal variation of b-value was utilized as earthquake precursor for large earthquake prediction (Gutenberg and Richter). Kagan (1997) analyzed earthquake data from 1965 to 1995 and concluded that the b-values are different between earthquakes on ocean ridges and in boundary zones of continents.

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Peer-review under responsibility of China University of Geosciences (Beijing).

http://dx.doi.org/10.1016/j.gsf.2017.04.006

Natural seismicity in California, Japan, Italy, and worldwide has been reported to exhibit increasing b-value moving from thrust to strike-slip to normal regimes (Schorlemmer et al., 2005; Gulia and Wiemer, 2010; Yang et al., 2012). Several studies have suggested a relation between differential stress and b-value (Tormann et al., 2012; Spada et al., 2013). Some studies also indicated depth dependence of b-value and suggested a systematic decrease of b-value with depth through the brittle zone prior to reaching the brittle-ductile transition zone (Gerstenberger et al., 2001; Spada et al., 2013). More recently, studies focus on associating b-value with parameters of subducting slabs; for example, b-value correlates positively with subducting plate age and depth of trench (Nishikawa and Ide, 2014), and b-value correlates linearly with slab pull force and with net reduction of plate interface normal force (Scholz, 2015), both of which also indicate a negative linear relation between b-value and differential stress.

Due to the significance of association of b-value with differential stress of tectonics, it is essential to explore the physical meaning and properties of b-value rather than considering it just as an empirical parameter of GR. Numerous studies have been devoted to develop models to simulate the GR power-law relations and to associate the b-value to fractal dimension of spatial distribution of earthquakes and physical property of raptures caused earthquakes. For example, Chen and Ji (1993) have proposed a new hierarchy model simulating seismicity which can produce fractal characteristics of seismicity. They derived explicit relation between the b-value of GR model and fractal dimension (D) of spatial distribution of epicenters. Instead of associating frequency-size distribution with spatial distribution, here we will characterize the size–frequency distribution from a singularity point of view. Singularity of a fractal model as proposed by the author refers to an index quantifying the degree of nonlinearity that anomalous amounts of energy release or mass accumulation confined to narrow temporal and spatial intervals (Cheng, 2016). We will first derive a new formulation based on GR to link b-value to the singularity index of density-scale power-law models that previously have been applied for interpreting other types of extreme events from a fractal density point of view. Then we will validate this model by applying it to the global database of earthquakes from 1964 to 2015 occurring on mid-ocean ridges, in subduction zones and in collision zones.

2. Methods

Here we will reformulate GR relation to associate the energy released by small fraction of large earthquakes which occur with small probability. Based on model (1) we can construct a truncated Pareto distribution as follows: assuming with a threshold E_0, there are a large number of earthquakes with energy release $E \geq E_0$ in a region, then the probability of an earthquake randomly selected from a region which has energy release $> E$ $(E \geq E_0)$ can be estimated by the following ratio of the number of large earthquakes over the total number of earthquakes of size $E \geq E_0$

$$P[>E] = \frac{N(>E)}{N(\geq E_0)} = \left(\frac{E}{E_0}\right)^{-\beta} \quad (3)$$

where $\beta = \frac{2}{3}$b. Accordingly the probability density function can be derived by taking first order derivative of the probability function (p) as follows (Newman, 2005)

$$p[E] = \beta E_0^{\beta} E^{-\beta-1} \quad (4)$$

From the probability density function (4) we can calculate the mean energy of earthquakes with size range between E_0 and E as follows

$$\overline{E}[<E] = \int_{E_0}^{E} \beta E_0^{\beta} E^{-\beta} \mathrm{d}E = \frac{\beta}{1-\beta} E_0 \left[\left(\frac{E}{E_0}\right)^{1-\beta} - 1\right] = \frac{E_0}{\Delta\alpha}\left[P(>E)^{-\Delta\alpha} - 1\right], \text{ if } \beta \neq 1 \quad (5)$$

and

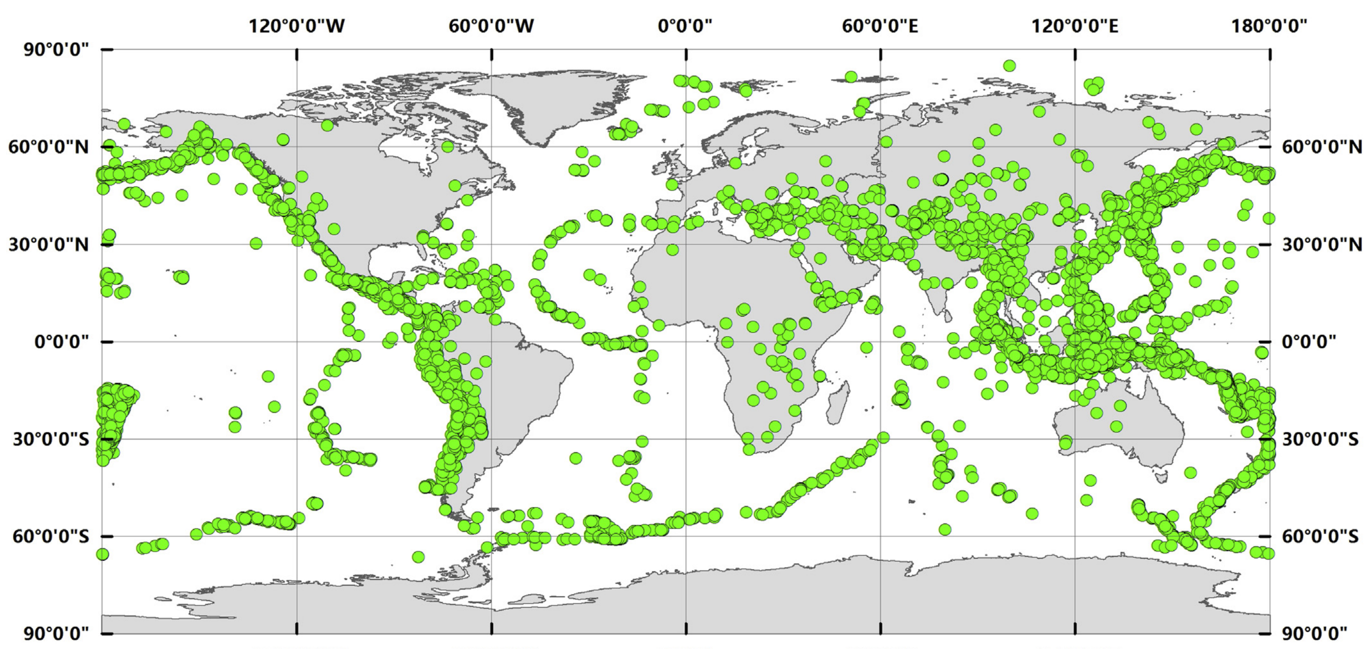

Figure 1. Locations of earthquakes with magnitude $M \geq 6$. Shaded regions represent continents and the dots for locations of earthquakes (Data retrieved from the ISC web site: International Seismological Centre, On-line Bulletin, http://www.isc.ac.uk, Internatl. Seismol. Cent., Thatcham, United Kingdom, 2015).

Please cite this article in press as: Cheng, Q., Sun, H., Variation of singularity of earthquake-size distribution with respect to tectonic regime, Geoscience Frontiers (2017), http://dx.doi.org/10.1016/j.gsf.2017.04.006