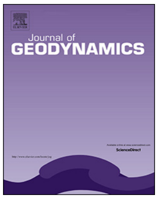
Journal of Geodynamics 119 (2018) 9–22

Contents lists available at ScienceDirect

Journal of Geodynamics

journal homepage: www.elsevier.com/locate/jog



Estimation and reduction of random noise in mass anomaly time-series from satellite gravity data by minimization of month-to-month year-to-year double differences

Pavel Ditmar, Natthachet Tangdamrongsub[1], Jiangjun Ran[2,*], Roland Klees

Delft University of Technology, Stevinweg 1, 2628 CN Delft, The Netherlands

A R T I C L E I N F O

Keywords:
Mass transport
GRACE
Tikhonov regularization
Variance Component Estimation
Tonlé Sap
Greenland Ice Sheet

A B S T R A C T

We propose a technique to regularize a GRACE-based mass-anomaly time-series in order to (i) quantify the Standard Deviation (SD) of random noise in the data, and (ii) reduce the level of that noise. The proposed regularization functional minimizes the Month-to-month Year-to-year Double Differences (MYDD) of mass anomalies. As such, it does not introduce any bias in the linear trend and the annual component, two of the most common features in GRACE-based mass anomaly time-series. In the context of hydrological and ice sheet studies, the proposed regularization functional can be interpreted as an assumption about the stationarity of climatological conditions. The optimal regularization parameter and noise SD are obtained using Variance Component Estimation. To demonstrate the performance of the proposed technique, we apply it to both synthetic and real data. In the latter case, two geographic areas are considered: the Tonlé Sap basin in Cambodia and Greenland. We show that random noise in the data can be efficiently (1.5–2 times) mitigated in this way, whereas no noticeable bias is introduced. We also discuss various findings that can be made on the basis of the estimated noise SD. We show, among others, that knowledge of noise SD facilitates the analysis of differences between GRACE-based and alternative estimates of mass variations. Moreover, inaccuracies in the latter can also be quantified in this way. For instance, we find that noise in the surface mass anomalies in Greenland estimated using the Regional Climate Model RACMO2.3 is at the level of 2–6 cm equivalent water heights. Furthermore, we find that this noise shows a clear correlation with the amplitude of annual mass variations: it is lowest in the north-west of Greenland and largest in the south. We attribute this noise to limitations in the modelling of the meltwater accumulation and run-off.

1. Introduction

The Earth's system is characterized by on-going large-scale mass transport. In most of land areas, it is associated with various hydrological processes. An exception are the polar regions, where the dominant contributors are ice sheets and Glacial Isostatic Adjustment (GIA).

An accurate quantification of large-scale masstransport is of major importance in various applications, including water management, climate science, and solid Earth geophysics. Satellite Gravimetry (SG) is a powerful tool to monitor large-scale mass transport. The first satellite mission suitable for that purpose – Gravity Recovery and Climate Experiment (GRACE) – was launched in 2002 (Tapley et al., 2004). In the first instance, SG data are used to compute time-series of the Earth's gravity field solutions. Typically, one solution per month is obtained. Each of them consists of a set of spherical harmonic coefficients complete to some maximum degree (usually between 60 and 120). After appropriate processing (see, e.g., Wahr et al., 1998; Ditmar, 2018), such solutions may yield a time-series of mass anomalies within a region of interest, i.e., the differences between the instantaneous amount of mass at (or near) the Earth's surface and the corresponding long-term mean value. Currently, the GRACE mission is not operational anymore, but its successor – GRACE Follow-On (GFO) – is scheduled for launch in early 2018 (Flechtner et al., 2014, https://gracefo.jpl.nasa.gov).

Mass anomalies extracted from SG data suffer from inaccuracies. A part of the error budget consists of random noise propagated from the original satellite observations via spherical harmonic coefficients. Such noise is not time-correlated and may be quite strong, especially if the

---

[*] Corresponding author.
E-mail address: j.ran@cuhk.edu.hk (J. Ran).
[1] Currently at: University of Newcastle, Australia.
[2] Currently at: State Key Laboratory of Geodesy and Earth's Dynamics, Institute of Geodesy and Geophysics, Chinese Academy of Sciences, Wuhan, China.

https://doi.org/10.1016/j.jog.2018.05.003
Received 2 August 2017; Received in revised form 13 March 2018; Accepted 11 May 2018

target region is small. The estimated mass anomalies may suffer also from systematic disturbances. For instance, various filters are typically used to reduce noise in spherical harmonic coefficients (Wahr et al., 1998; Han et al., 2005; Wouters and Schrama, 2007; Swenson and Wahr, 2006; Kusche, 2007; Klees et al., 2008; Siemes et al., 2013). Unfortunately, filters also distort the signal of interest, introducing among others leakage errors.

The random and systematic errors mentioned above may complicate the usage of SG-based mass anomaly estimates in practice. For instance, these errors make it more problematic to estimate the quality of a geophysical model describing mass transport of a certain type when SG is used as a source of independent information. This is because the differences between the geophysical model and SG-based estimates will be contaminated by errors in the latter estimates themselves. This may be particularly harmful if errors in SG-based estimates are comparable to or exceed errors in the geophysical model.

With this article, we present a novel procedure that allows for: (i) quantifying the level of random noise in a mass anomaly time-series based on SG data; and (ii) reducing this level. The basic properties of the proposed procedure are as follows:

- It is based on the Tikhonov regularization concept (Tikhonov and Arsenin, 1977) and does not require an explicit parameterization of the signal in the time domain, which makes the procedure very flexible.
- A new variant of the regularization functional is proposed, which minimizes the month-to-month year-to-year double differences in order to keep seasonal variations and linear trends (the dominant features of many mass transport processes) untouched, so that the bias introduced by the regularization is reduced.
- Known stochastic properties of random noise (e.g., time-dependent standard deviation or full error variance-covariance matrix) can be accounted for in the statistically optimal way.
- The optimal regularization parameter is computed by Variance Component Estimation (VCE) (Koch and Kusche, 2002), which makes the procedure not only flexible, but also fully automatized.
- VCE allows also for a re-estimation of the random noise level in the original SG-based estimates.

The ability of the procedure to quantify the level of random noise in a mass anomaly time-series from the time-series itself makes it particularly valuable when SG is used for the validation of a geophysical model. Knowledge of this level allows for a quantification of the contribution of random noise in SG-based estimates to their differences with respect to the time-series subject to validation. Then, it is easy to estimate the Standard Deviation (SD) of remaining noise, which is composed of systematic errors in SG estimates and noise in the geophysical model assuming that remaining noise is not correlated with random noise in SG estimates. This opens the door for the quantification of noise in the geophysical model alone (since the contribution of systematic errors in SG estimates can be assessed by, e.g., a numerical experiment).

The proposed procedure has been already successfully used in a number of studies: to assess the performance of a novel variant of a so-called mascon approach in the context of Greenland Ice Sheet monitoring with SG (Ran et al., 2018); to calibrate the error covariance matrices of degree-1 and C_{20} spherical harmonic coefficients estimated from a combination of GRACE-based monthly solutions and an ocean bottom pressure model (Sun et al., 2017); as well as to demonstrate the added value of a novel technique for GRACE data processing by considering the estimated mass anomalies in Mississippi River basin and in Greenland (Guo et al., 2018). In this article, we present an in-depth analysis of the proposed technique, including an open discussion of its strong points and limitations. We focus on two geographical areas as representative examples. The first one is the Tonlé Sap basin (Cambodia), which is subject to large seasonal and inter-annual mass variations of hydrological origin. The other area is Greenland, where a combination of snowfalls and ice mass losses results in strong seasonal mass variations coupled with large negative long-term trends. The two examples were deliberately chosen to demonstrate that the proposed methodology has a broad spectrum of potential applications. Among others, we discuss how the aforementioned “remaining noise” can be quantified and how this information can be used to know more about a mass anomaly time-series alternative to the SG-based one. In addition, we isolate the “remaining noise” in the differences between *regularized* SG estimates and the alternative time-series. This allows us to quantify the level of random noise in SG estimates after regularization and, therefore, to assess how efficiently that noise is damped by the proposed procedure.

The structure of the article is as follows. Section 2 contains a description of the proposed regularization procedure. In Section 3, we apply the developed procedure to mass anomaly time-series based on simulated and real GRACE data. Among others, we discuss in detail how the SD of “remaining noise” and the reduction of random noise by regularization can be quantified (Section 3.1.2). Furthermore, realistic numerical simulations are conducted in order to support real data processing and make a comprehensive assessment of performance of the proposed regularization scheme. Section 4 contains a discussion and conclusions.

2. Theory

Mass anomaly time-series $H_i^{(\mathrm{obs})}$ based on SG data may contain gaps and strong noise. The proposed technique allows for a quantification and reduction of the noise level, as well as for filling in data gaps, if they are present. To that end, the Tikhonov regularization concept (Tikhonov and Arsenin, 1977) is used. To simplify the presentation of the method, we assume that the regularized mass anomaly time-series is a continuous function $\hat{H}(t)$, where t is time in years. The corresponding equations for discrete time-series are provided in Appendix A. In the actual implementation of the proposed technique, the discretization of the original and regularized time-series is always one month.

We postulate that the regularized time-series $\hat{H}(t)$ minimizes the penalty functional

$$\Phi[H] = \sum_i (H(t_i) - H_i^{(\mathrm{obs})})^2 + \alpha\Omega[H], \tag{1}$$

where t_i is the time of the ith observation, α is the regularization parameter, and $\Omega[H]$ is the regularization functional. The latter depends on the function $H(t)$ and its derivatives up to a given order. For simplicity, we assume here that noise in the input data is white. A generalization to arbitrary Gaussian noise is straightforward (see Appendix A).

The highest order of the derivatives of $H(t)$ used in the definition of the regularization functional defines the order of that functional. A commonly-used Tikhonov regularization functional is the zero-order functional

$$\Omega[H] = \int (H(t))^2\,\mathrm{d}t, \tag{2}$$

which requires that the target function $\hat{H}(t)$ is as close to zero as possible. As an alternative, the first-order functional

$$\Omega[H] = \int (H'(t))^2\,\mathrm{d}t \tag{3}$$

(where $H'(t)$ is the time-derivative of $H(t)$) is used frequently. This functional tries to make the unknown function the smoothest possible one. In the context of GRACE data processing, a somewhat similar idea was applied in the computation of mascon solutions (see, e.g., Luthcke et al., 2006, 2013). Both zero- and first-order functionals inevitably bias the solution, since they penalize all signals (an exception is a constant, which is not penalized by the first-order functional). This makes their application to mass anomaly time series sub-optimal.