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A method for determining the regularization parameter and the relative weight ratio of the seismic slip distribution with multi-source data

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ABSTRACT

In this paper, a combined approach consisting of the U-curve method and discriminant function minimization method is proposed to determine the regularization parameter and relative weight ratio of observed multi-source data for a joint inversion of the seismic slip distribution. The proposed method uses the U-curve method to determine the optimal regularization parameter of observed data (GPS and InSAR) through individual inversion and then uses the discriminant function minimization method to determine the relative weight ratio between GPS and InSAR data. Simulation experiment results show that the method proposed in this paper is better than the variance component estimation method; and the joint U-curve and variance component estimation method, in terms of the root mean square error. In addition, the inversion results of the L'Aquila and Meinong earthquakes further prove the feasibility and advantage of the proposed method.

1. Introduction

Geodetic inversion represents one of the core issues of geodesy in the geoscience field (Xu, 2001). Geodetic-geophysical joint inversion has developed from the use of single datasets to a variety of geodetic data types and multi-source data (involving geodetic data, seismic wave data, geophysical data and geological data) (Xu et al., 2016). Geodetic-geophysical joint inversion is an important method for studying geodynamics and the movement of tectonic blocks and for exploring the relationship between crustal movements and earthquakes. In addition, the key problem in joint inversion is the determination of the relative weight ratios between different types of data. A successful joint inversion must have a reasonable relative weight ratio. Consequently, research on the relative weight ratio has received intense interest in joint inversion (Wang et al., 2012).

InSAR data and GPS data are effective for an inversion of the seismic slip distribution, and these data have consequently extensively studied and applied by scholars because of their wide coverage and high accuracy. InSAR uses the strength and phase information of radar signals to obtain line-of-sight information of the Earth's surface. Radar signals can work in all kinds of weather conditions. The InSAR technique can obtain a large, high-precision digital elevation model and complement elevation data in remote areas with harsh climates (Xu, 2001). DInSAR technology can be employed to detect surface deformation at the centimeter and even millimeter scales (Carnec and Fabriol, 1999). In addition, due to its advantageous capacity to provide a high spatial coverage and its relatively low cost, radar interferometry has been used throughout seismic research in recent years (e.g., Wang et al., 2017a,b; Zhang et al., 2010; Liu et al., 2005). However, InSAR technology is very sensitive to atmospheric influences (i.e., tropospheric and ionospheric delays), satellite orbit errors, surface conditions and time variation correlations, and none of which can be eliminated by SAR data alone. Meanwhile, the precise positioning of GPS can determine the accurate locations and elevation changes of discrete ground points, and can also accurately determine the parameters of the ionosphere and troposphere. The GPS technique is the most accurate, convenient and practical method for studying crustal deformation at present (Xu, 2001). However, InSAR and GPS technologies can complement each other and guarantee temporal and spatial data continuity (Xu, 2001). Therefore, we study an inversion of the seismic slip distribution by combining InSAR data with GPS data.

To avoid the rank deficiency of the coefficient matrix and guarantee the smoothness between patches of the fault plane, The Laplace operator is usually introduced for constraints in seismic slip distribution inversion (Jónsson et al., 2002). This paper combines InSAR data with GPS data to carry out seismic slip distribution inversion while focusing on a determination of the relative weight ratio between InSAR data and GPS data and the optimal regularization parameter (i.e., smoothing

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factor). To determine the relative weight ratio of the seismic slip distribution of multi-source data, Fukuda and Johnson (2008) derived a Bayesian inversion methods and Fukuda and Johnson (2010) used the F-J method. Furthermore, Xu et al. (2016) and Fan et al. (2017) proposed a variance component estimation method. These three methods all treat the regularization parameters as the weights of virtual observation(abbreviation: VO) data, and thus, the regularization parameters and the relative weight ratio between observed data can be obtained at the same time. (during an inversion of the slip distribution, the fault surface needs to be discretized into homogeneous or inhomogeneous small fault patches; Moreover, it is necessary to apply a smoothing constraint to the dislocations along fault patches to avoid oscillations in the slip distribution solution, and the observed data in the imposed constraints are regarded as virtual observation data). The first two methods are complicated and have a low computational efficiency. Moreover, the goal of determining the relative weight ratio in the variance component estimation method is to unify the variance in the unit weight of the observed data during the process of seismic slip distribution inversion. However, the variance component estimation method may return a negative variance (Amiri-Simkooei, 2016). For the selection of regularization parameters, existing studies use various methods, namely, the generalized cross verification method (Golub, 1979) and the L-curve method (e.g., Hansen and O'Leary, 1993, Wang and Ou, 2004); in addition to adaptive Tikhonov regularization with multiple regularization parameters (Wang et al., 2016). Nevertheless, the GCV method can theoretically obtain better regularization parameters, but the curve drawn by the GCV function is sometimes too gentle, and thus, it is difficult to locate the optimal regularization parameter (Lu and Wang, 2015). Meanwhile, the adaptive regularization method has some advantages over the conventional method, but its calculation process is more complex than that of traditional method (Wang et al., 2016). Moreover, there are many problems in the L-curve solution, such as an over-reliance on the data fit, and the convergence of the solution process (Xu, 2010; Yuan et al., 2010).

Considering the above problems, this paper proposes the U-curve method (e.g., Krawczyk-Stańdo and Rudnicki, 2007; Reginska, 1996; Chamorroservent et al., 2011; Arnrich et al., 2014; Zhong et al., 2014; Lu and Wang, 2015) to determine the optimal regularization parameter, and the relative weight ratio between observed InSAR and GPS data obtained by the discriminant function minimization method (abbreviation: UDFM) (e.g., Xu et al., 2009; Yu, 2016). This paper uses the U-curve method to obtain the optimal regularization parameters of the observed data (GPS and InSAR) through individual inversion and then uses the discriminant function minimization method to determine the relative weight ratio between the GPS data and InSAR data. Several groups of simulation experiments are carried out, and the following three methods are used for seismic slip distribution inversion: (1) the UDFM method; (2) the variance component estimation (abbreviation: VCE) method; (3) the U-curve with VCE (abbreviation: UVCE) method. The inversion parameters obtained by using the above three methods are compared. This paper also applies the above methods to the L'Aquila Mw6.3 earthquake in 2009 and the Meinong, Taiwan Mw6.4 earthquake in 2016, compares the results of these three methods, and then explores the feasibility and advantage of each method.

2. The basic principles of the U-curve method for determining the regularization parameter

2.1. The equations for seismic slip distribution inversion

In this paper, we use a non-negative least squares algorithm for the linear inversion. The relation between the seismic slip distribution displacement and the fault slip can be expressed as

$$\begin{cases} \boldsymbol{d} = \boldsymbol{Gm} + \varepsilon \\ \boldsymbol{m} \geq 0 \end{cases} \tag{1}$$

where $\boldsymbol{d}$ represents the line-of-sight variable observed by InSAR or the shape variable of three directions observed by GPS; $\boldsymbol{G}$ is the corresponding Green's matrix; $\boldsymbol{m}$ is the seismic slip parameter; and ε is the observation error of InSAR data or GPS data.

To avoid the rank deficiency of the coefficient matrix and guarantee the smoothness between each block of the fault plane, the Tikhonov regularization method (Tikhonov, 1963) is used to solve the slip distribution. In addition, the Laplace operator, which is successfully used for smoothness constraints of fault planes at present. Thus, we obtain

$$\boldsymbol{Hm} = \boldsymbol{0} \tag{2}$$

where the $\boldsymbol{0}$ matrix is the virtual observation matrix; $\boldsymbol{H}$ is the Laplacian smoothing matrix; and $\boldsymbol{R} = \boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}$ is considered a regularization matrix. According to the regularization theory proposed by Tikhonov (1963), the estimation criteria can be expressed as

$$\|\boldsymbol{Gm} - \boldsymbol{d}\|_{\boldsymbol{P}}^2 + \alpha\Omega(\boldsymbol{m}) = \|\boldsymbol{Gm} - \boldsymbol{d}\|_{\boldsymbol{P}}^2 + \alpha\boldsymbol{m}^T\boldsymbol{Rm} = \min \tag{3}$$

where α is the regularization parameter corresponding to the inversion of InSAR or GPS data; $\Omega(\boldsymbol{m})$ is a stable functional; $\boldsymbol{p}$ is the weight matrix of the observation vector; ||•|| is the two norm of •; and the regularization parameter α in this paper is determined by the U-curve method.

2.2. Basic principle of the U-curve method

With Eqs. (1) and (2), the seismic slip distribution solution can be expressed as

$$\boldsymbol{m} = (\boldsymbol{G}^{\mathrm{T}}\boldsymbol{pG} + \alpha\boldsymbol{R})^{-1}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{pd} \tag{4}$$

Eq. (4) shows that the seismic slip distribution solutions varies with the regularization parameter α. Therefore, choosing a reasonable regularization parameter is very important for an inversion of the seismic slip distribution.

The U-curve method is similar to the L-curve method when determining the regularization parameter. The L-curve method obtains a set of $\|\boldsymbol{Gm} - \boldsymbol{d}\|^2$ and $\|\boldsymbol{Rm}\|^2$ values based on different α values. $\|\boldsymbol{Rm}\|^2$ is set as the ordinate and $\|\boldsymbol{Gm} - \boldsymbol{d}\|^2$ is set as the abscissa; a fitting of their values can generate a curve, shaped like an "L". The α value with the highest curvature represents the optimal regularization parameter. Thus, the accuracy of the L-curve method depends on the fitting degree of the $\|\boldsymbol{Gm} - \boldsymbol{d}\|^2$ and $\|\boldsymbol{Rm}\|^2$ data. By defining the $U(\alpha)$ function, the U-curve method obtains the $U(\alpha) - \alpha$ curve based on the values of α. The objective of the U-curve criterion for selecting the regularization parameter is to choose a parameter for which the curvature attains a local maximum close to the left-hand vertical part of the U-curve (e.g., Krawczyk-Stańdo and Rudnicki, 2007; Chamorroservent et al., 2011; Arnrich et al., 2014; Zhong et al., 2014; Lu and Wang, 2015). The U-curve method can be defined as follows.

$$U(\alpha) = \frac{1}{\|\boldsymbol{Gm} - \boldsymbol{d}\|^2} + \frac{1}{\|\boldsymbol{Rm}\|^2} \tag{5}$$

According to Eq. (5) (Reginska, 1996; Zhong et al., 2014, Lu and Wang, 2015), we can obtain $\|\boldsymbol{Gm} - \boldsymbol{d}\|^2 = \sum_{i=1}^{r} \frac{\alpha^4\beta_i^4\theta_i^2}{(\sigma_i^2 + \alpha^2\beta_i^2)^2}$ and $\|\boldsymbol{Rm}\|^2 = \sum_{i=1}^{r} \frac{\sigma_i^2\beta_i^2\theta_i^2}{(\sigma_i^2 + \alpha^2\beta_i^2)^2}$, where $\boldsymbol{\theta} = \boldsymbol{u}^{\mathrm{T}}\boldsymbol{d}$; $\boldsymbol{u}$ is the left-hand singular value matrix of the coefficient matrix $\boldsymbol{G}$ after singular value decomposition; $\sigma_1 \geq \sigma_2 \ldots \geq \sigma_r > 0$ are the singular values of $\boldsymbol{G}$; and $\beta_1 \geq \beta_2 \ldots \geq \beta_r > 0$ are the singular values of the Laplacian matrix. Therefore, because there is no curve fitting and fewer calculations are required, the U-curve method is better than the L-curve method.