International Journal of Sediment Research 33 (2018) 126–136

Contents lists available at ScienceDirect

International Journal of Sediment Research

journal homepage: www.elsevier.com/locate/ijsrc

Original Research

Bed-load transport rate based on the entrainment probabilities of sediment grains by rolling and lifting

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ARTICLE INFO

Article history:
Received 30 January 2017
Received in revised form
18 August 2017
Accepted 29 December 2017
Available online 8 January 2018

Keywords:
Sedimentation
Bed load
Entrainment probability

ABSTRACT

A function for the bed-load sediment transport rate is derived. This function is obtained by using the entrainment probabilities of the rolling and lifted sediment grains, and by introducing two travel lengths, respectively. The predictions from the new bed-load function agree well with experimental results over the entire experimental range and show significant improvement over the commonly used formula for the bed-load transport rate. The new function shows that, in terms of contributing to the bed-load transport rate, the total entrainment probability of the sediment grains is a weighted summation of those for the lifted and rolling grains, rather than a simple addition of the two. The function is also used to predict the total entrainment probability, saltation length, and the bed layer thickness at a high bed-load transport rate. These predictions all agree well with the experimental results. It is found that, on average, the travel length for the rolling sand grains is about an order of magnitude less than that of the lifted grains.

1. Introduction

Sediment transport is a part of the fluvial processes and is closely related to the morphological changes. In general, there are two forms of sediment transport, which are called the bed load and suspended load. In bed load, the sediment grains may roll, slide, or travel in a succession of low jumps, termed saltation, close to the bed surface from where they may temporarily leave (Dey, 2014).

It is generally believed that the bed load is closely related to the entrainment of the sediment grains being pickup by the fluid motion near the bed surface when the hydraulic force acting on the grain is larger than the grain resistance to motion. Here pickup means that the sand grains have been moved relative to the stationary bed surface. The movement can be in the modes of rolling, sliding, bouncing, and lifting. Over the years, the pickup of sediment grains by the fluctuating motion of the flow near the bed surface has been extensively investigated theoretically, experimentally, and numerically (Agudo et al., 2014; Apperley & Raudkivi, 1989; Cao, 1997; Celik et al., 2010; Dwivedi et al., 2010, 2011; Einstein & El-Sammi, 1949; Grass, 1970, 1983; Hofland et al., 2005; Jain, 1991; Leighly, 1934; McEwan et al., 2004; Nelson et al., 1995, 2001; Paiement-Paradis et al., 2003; Paintal, 1971; Papanicolaou et al., 2001; Schmeeckle et al., 2007; Sumer et al., 2003; Sutherland, 1967; Valyrakis et al., 2010, 2013; Vollmer & Kleinhans, 2007; Zanke, 2003). In these investigations, the dependence of the incipient motion on the magnitude of instantaneous velocity, the roughness and irregularity of the bed surface texture and morphology, average bursting period and area, impulse (instantaneous forces and duration), and flow power has been studied.

Given the complexity of the sediment transport in the bed layers where complex interactions exist between a large amount of sediment grains and interactions between solid grains and the turbulent flows, it is difficult to predict the bed-load transport rate analytically based on the incipient motion of individual sand grains. So far, there are mainly two approaches in deriving the bed-load formula theoretically. One is the deterministic approach such as those by Bagnold (1954) and Yalin (1977). The second is the probabilistic approach such as those by Einstein (1942, 1950), Engelund and Fredsøe (1976), Wang et al. (2008), and Zhong et al. (2012). The estimated bed-load transport rate from many of the bed-load formulae in general compare well with the experimental results in a limited range and it has been a challenge to develop formulae to predict results with good agreement covering the full experimental range.

In this study, the probabilistic approach is taken since all the investigations on the instantaneous velocity, average bursting

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http://dx.doi.org/10.1016/j.ijsrc.2017.12.005

Nomenclature

A	frontal area of a sediment grain (m^2)
A_1	surface area of the bed surface (m^2)
B	constant
B_L	threshold for a sediment grain to be lifted (m/s)
B_R	threshold for a sediment grain to begin rolling (m/s)
$\overline{C}$	average volumetric concentration of sediment grains in the bed layer
C_a	sediment volumetric concentration at the reference location
C_b	sediment volumetric concentration at the bed surface
C_D	drag coefficient
C_L	lift coefficient
C_v	sediment volumetric concentration in the suspended sediment region
d	diameter of the sediment grain (m)
F_D	hydraulic drag force acting on a grain (N)
F_L	hydraulic lift force acting on a grain (N)
$f(u)$	probability distribution function of the streamwise velocity fluctuation
g	acceleration of gravity (m/s^2)
H	channel depth (m)
k_s	equivalent roughness of the sediment grain (m)
L	travel length of a sediment grain (m)
L_L	travel length of a sediment grain by lifting (m)
L_R	travel length of a sediment grain by rolling (m)
L_x	adaptation length of a sediment grain (m)
$\dot{m}$	mass flow rate of sediment grains from the bed surface (kg/s)
$\dot{m}_L$	mass flow rate of sediment grains from the bed surface by lifting (kg/s)
$\dot{m}_R$	mass flow rate of sediment grains from the bed surface by rolling (kg/s)
n	number of sediment grains in motion
n_L	number of sediment grains in motion by lifting
n_R	number of sediment grain in motion by rolling
P_E	total entrainment probability of the sediment grain as according to Eq. (40)
P_L	probability of the lifted sediment grains
P_R	probability of the rolling sediment grains
P_T	total probability of the sediment grain in motion as according to Eq. (24)
q_b	total transport rate of the bed load (m^2/s)
q_{bL}	transport rate of the bed load by lifting (m^2/s)
q_{bLR}	transport rate of the bed load by rolling (m^2/s)
q_s	transport rate of the suspended load (m^2/s)
q_t	total transport rate of the sediment grains (m^2/s)
R_L	moment arm length for the lifting force (m)
R_p	Reynolds number of sand grains
R_R	moment arm length for the drag force (m)
R_W	moment arm length for the gravity force (m)
u	instantaneous streamwise velocity (m/s)
$\overline{u}$	average streamwise velocity (m/s)
u_b	average streamwise velocity acting on a sediment grain (m/s)
u_*	wall shear velocity (m/s)
W	submerged weight of a sediment grain (N)
w_s	terminal fall velocity of a sediment grain (m/s)
y	distance from the bed surface (m)
y_0	distance where local mean velocity is zero (m)
y_a	the reference location in using the Rouse profile for the suspended load (m)
y_b	location where the average velocity is u_b (m)
α	the ratio of the travel lengths of the sediment grains by rolling and that by lifting
β	time constant
γ	friction angle
Δ	submerged relative density of the sediment grains
δt	time scale of the sediment grain in motion (s)
δ_s	thickness of the bed layer (or sheet flow layer) (m)
Φ	non-dimensional transport rate of the bed load
κ	von Karman constant
λ	constant
λ_1	saltation length constant
λ_2	rolling length constant
λ_b	saltation length (m)
θ	Shields parameter
θ_c	critical Shields parameter
ρ_f	density of the fluid (kg/m^3)
ρ_s	density of the sediment grains (kg/m^3)
σ_u	standard derivation of the streamwise velocity fluctuation (m/s)

period and area, impulse (instantaneous forces and duration), and flow power show that the pickup of the sediment grains from the bed surface is related to the instantaneous velocity (mean plus fluctuations) of the flow. Given that the flows in nearly all open channels with sediment transport of engineering interests are turbulent, and the instantaneous flows (or pressure fluctuations, bursting duration and area, and/or impulse and energy) near the bed surface will be random processes, and, thus, a probabilistic approach is appropriate.

Einstein (1942, 1950) is the pioneer in the development of a bed-load transport model based on the probabilistic concept. By considering the probability of the dynamic lift on sediment grains being larger than their submerged weight, Einstein was able to theoretically derive a pickup function. By combining this function with a travel length of the moving grains near the bed surface and an assumed time scale for the grains to be lifted, Einstein (1950) was able to derive a bed-load formula (with many parameters determined by experimental results). The advantage of using the probabilistic approach over that of a deterministic one is that a threshold criterion for the initiation of the grain movement is avoided since it is always a difficult proposition to define (Dey, 2014) and it may depend not only on the Reynolds numbers but also on factors such as the angle of repose of the bed sediment (Dey, 1999) and the mode of sand movement (sliding or rolling) (Ali & Dey, 2016). The pickup probability function of Einstein (1950) was based on the instantaneous velocity instead of the time-averaged velocity. Engelund and Fredsøe (1976) suggested that the pickup probability is related to both the dimensionless bed load and the dimensionless wall shear stress when considering the motion of an individual sediment grain contributing to the bed load. They determined their probability function by empirically fitting experimental data. Sun and Donahue (2000) derived a bed-load formula for the arbitrary size fraction of non-uniform sediment grains based on a method of combining stochastic processes with mechanics. They considered both the probability of fractional incipient motion and that from the average velocity of grain motion.

Cheng and Chiew (1998) investigated the effect of the lift coefficient on the pickup probability in a hydraulically rough flow and concluded that the pickup probability is about 0.6% when compared to the Shields criterion (Shields, 1936) for sediment transport. In Cheng and Chiew (1998), the probability distribution