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RESEARCH PAPER

A wellbore creep model based on the fractional viscoelastic constitutive equation

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Abstract: To simulate the evolution of wellbore creep accurately, predict and prevent severe accidents such as borehole wall sloughing, casing collapse and sticking of the drill, based on previous studies, the springpot element was introduced into the classical element model and the creep compliances of the fractional constitutive models were deduced. The good fitting effect of fractional constitutive model was verified. The study shows the fractional constitutive model can simulate creep with high accuracy and less input parameters, and the physical significance of the input parameters are clearer. According to the correspondence principle of viscoelastic theory, a wellbore creep model including drilling and killing processes was built up. By adjusting the value of fractional orders, the model can transform between the models of ideal elastic material and standard solid, which implies the classical wellbore shrinkage model based on standard solid model and ideal elastic model are just special cases of this model. If the fractional order is adjusted, the creep curve will change asymmetrically, which can be can be regulated by the speeding up of the transient creep and lowering the rate of steady creep, which can not be accomplished by adjusting one parameter in the classical models. The fractional constitutive model can fit complicated non-linear creep experiment data better than other models.

Key words: viscoelastic constitutive equation; fractional model; springpot element; wellbore shrinkage; rock creep; wellbore stability

Introduction

Wellbore shrinkage often takes place in the salt rock, coal and shale formations with apparent viscoelastic behaviors[1–4], which could lead to severe accidents such as borehole wall sloughing, casing collapse and sticking of the drilling tool[5–7]. The rock creep has a stronger influence on the development of coalbed methane, shale gas and tight gas/oil than on conventional reservoirs[8–9]. Rock viscoelastic research has shown that the confirmation and application of viscoelastic constitutive equation is the key to accurate simulation of the creep phenomenon[10–11]. In recent years, fractional calculus has made major progress. With the Caputo definition of fractional calculus being put forward, the initial condition of fractal dimension is avoided[12–13]. Hence, fractional constitutive equation is widely used in the simulation of viscoelastic materials due to its clear physical significance, low attenuation rate at later stage and high fitting efficiency[14–16]. Based on the previous researches, the creep and relaxation properties of springpot element were studied, and the springpot element was introduced into the classical viscoelastic constitutive equation. The fitting efficiencies of classical and fractional viscoelastic constitutive equations were compared. In conjunction with the correspondence principle of viscoelastic theory, fractional viscoelastic constitutive equation was used in the simulation of wellbore creep to guide oilfield engineering.

1. Springpot element

Viscoelastic constitutive equations include three categories: empirical models, mechanism-based models and element models[17]. Empirical models are achieved by fitting real experimental data, and the fitting efficiency is quite good since there is no limitation on fitting function selection. But the physical significances of the input parameters are not very clear, so the time and dimension extrapolation cannot be guaranteed. Mechanism-based models take the influences of damages and micro fractures into consideration, but some of the input parameters are defined by different engineers, which may affect the final simulation results. Element models have clearer physical significance, and have the base of time and dimension extrapolation. However, for most viscoelastic materials, in order to reach the simulation precision, too much elements are needed with higher workload[18]. With the advancement of fractional models, many researchers believe the attenuation rate of fractional viscoelastic model is much

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slower, so the complex viscoelastic behaviors can be simulated with fewer elements[19–20].

Fractional element is called soft element or Abel pot in some of the references[21], in this paper it is referred to as springpot according to references [19] and [20]. Its name should be combined by spring and dashpot since it has both of the properties of spring element and dashpot element.

The constitutive equation of springpot is:

$$\sigma(t) = \eta D^{\alpha} \varepsilon(t) \tag{1}$$

where we use the Caputo definition for fractional differential operator[12–13]:

$$D^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f'(\tau)}{(t-\tau)^{\alpha}} \mathrm{d}\tau$$

When stress is constant, we can get the creep model of springpot:

$$\varepsilon(t) = \frac{\sigma_0}{\eta} \frac{t^{\alpha}}{\Gamma(1+\alpha)} \tag{2}$$

When strain is constant, we can get the relaxation model of springpot:

$$\sigma(t) = \eta \varepsilon_0 \frac{t^{-\alpha}}{\Gamma(1-\alpha)} \tag{3}$$

Creep and relaxation curves at different fractional orders are shown in Fig. 1.

The fractional order α is the specific parameter in equations (1) - (3). When α approaches to 0, the properties of springpot is much similar to ideal elastic material. When α approaches to 1, the properties of springpot become more and more close to viscous fluid. When α changes from 0 to 1, viscoelastic behaviors with creep or relaxation characteristics can be simulated

Fig. 1 shows the creep curves and relaxation curves at different fractional orders differ widely. For example, taking the curve at fractional order of 0.5 as a dividing line, we can find the denseness of curves in high attenuation rate area is much less than that in low attenuation rate area. Hence, the springpot element is more suitable for simulating materials more similar to elastic solid, such as rock.

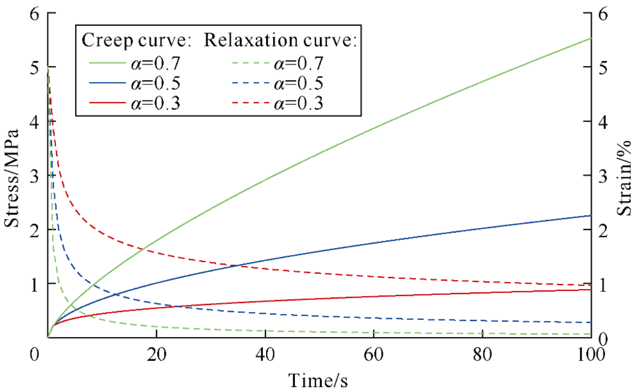

Fig. 1. Creep and relaxation curves of srpingpot at different fractional orders. The initial stress is 1 MPa, the initial strain is 1 and the fractional consistency coefficient is 5 MPa·s$^{\alpha}$.

2. Correspondence principle of viscoelastic theory

In viscoelasticity, when the function of transient stress with time is step form, the transient strain can be expressed as:

$$\varepsilon(t) = J(t)\sigma_0 \mathrm{H}(t) \tag{4}$$

When the function of transient stress is more complex, it can be transformed into convolution form through increment model[22]:

$$\varepsilon(t) = \sigma(t)J(0) + \sigma(t) * \mathrm{d}J(t) \tag{5}$$

where $\sigma(t) * \mathrm{d}J(t) = \int_0^t \sigma(\tau)\mathrm{d}J(t-\tau)$

Introducing summation convection into the three dimensional stress condition, the constitutive equation can be described by bulk modulus and shear modulus:

$$e_{ij} = \frac{S_{ij}}{2G} + S_{ij} * \mathrm{d}J_{\mathrm{e}} \tag{6}$$

$$\varepsilon_{ii} = \frac{\sigma_{ii}}{3K} + \sigma_{ii} * \mathrm{d}J_{\mathrm{v}} \tag{7}$$

According to the correspondence principle of viscoelastic theory, when the position and type of boundary conditions don't change, the only difference of elastic and viscoelastic equations in Laplace space is constitutive equation. The elastic constitutive equation in Laplace space is:

$$\hat{S}_{ij} = 2\hat{G}\hat{e}_{ij} \tag{8}$$

$$\hat{\sigma}_{ii} = 3\hat{K}\hat{\varepsilon}_{ii} \tag{9}$$

Viscoelastic constitutive equation in Laplace space could be represented by equation (6) and (7)[23].

$$\hat{e}_{ij} = s\hat{J}_{\mathrm{e}}\hat{S}_{ij} \tag{10}$$

$$\hat{\varepsilon}_{ii} = s\hat{J}_{\mathrm{v}}\hat{\sigma}_{ii} \tag{11}$$

Substituting equations (10) and (11) into equations (8) and (9) respectively, the solution of viscoelastic problem in Laplace space is:

$$\hat{G} = \frac{1}{2s\hat{J}_{\mathrm{e}}} \tag{12}$$

$$\hat{K} = \frac{1}{3s\hat{J}_{\mathrm{v}}} \tag{13}$$

Stress difference and axial strain are often used to calculate the Young's modulus, the transformation form also can be deduced similarly in practical rock mechanics test:

$$\hat{E} = \frac{1}{s\hat{J}_{\mathrm{a}}} \tag{14}$$

3. Fractional creep constitutive equation and case study

In regular element models, one element just introduces one undetermined parameter, but springpot element will introduce two parameters. Hence, when comparing different constitutive equations, the number of undetermined parameters should be regarded as index. Kelvin model, standard solid model and Burgers model are the main models used for simulating transient and steady creep classical viscoelastic constitutive models[22]. They are shown in Fig. 2.