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RESEARCH PAPER

Applicable conditions and analytical corrections of plane strain assumption in the simulation of hydraulic fracturing

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Abstract: Plane stain assumption is widely used in the simulation of the distribution of stress and displacement around hydraulic fractures. According to the comparison of the solutions of elliptical fracture and plane stain fracture, the applicable conditions of plane strain assumption were discussed and the correction method was put forward. On the basis of the past research, a classical stress and displacement solution around a flat elliptical fracture was deduced, fulfilled and corrected. In comparison with the plain strain fracture solution, simulating results show that if taking plane strain assumption into consideration in the fracture height profile, the difference between the elliptical fracture solution and plane strain solution is negligible when the ratio between fracture length and fracture height is larger than 10, and the above critical value could be relaxed to 5 in some simulations which normal stress perpendicular to fracture face plays a decisive role. The plane strain fracture solution is accurate enough when the above critical value is satisfied, the correction of the plane strain fracture solution is needed when the length-height ratio is small. The correction charts of the additional initial stress of the horizontal well with an open-hole and the width of a single fracture were drawn under different length-height ratios. The fracture width of pseudo three dimension propagation models is easier to be modified by the correction charts.

Key words: hydraulic fracturing; plane strain; elliptical fracture; analytical model; correction chart

Introduction

Plane problem is one kind of ideal problems simplified from elasticity[1]. When the dimension in the direction of x is much shorter than that in the directions of y and z, all external forces are perpendicular to the x axis, the shape of the profiles and the the external forces are not changing along the variation of *x axis*, this kind of question can be regarded as plane stress one. Conversely, when the dimension in the direction of x is much larger than that in the directions of y and z, external forces are perpendicular to the x axis, the shape of the profiles and the external forces are not changing with the variation of x, this kind of question can be regarded as plane strain one. The plain strain assumption has been widely adopted in the classical 2D model[2] or pseudo 3D (P3D) model in simulation of hydraulic fracturing[3], but whether the plain strain assumptions are appropriate has not been studied systematically. Simulating the distribution of stress and displacement around the hydraulic fracture accurately is of great significance for refracturing design[4–5], modeling hydraulic fracture diversion and propagation[6–8], optimizing the interval of staged fracturing[9–12], etc. Therefore, on the basis of previous research, the classical stress and displacement field expression around the flat elliptical fracture has been deduced, supplemented and corrected, by comparing the solutions of elliptical fracture and plane stain fracture, the applicable conditions of plane strain assumption have been analyzed and the plane stain solution has been corrected in this study.

1. Physical and mathematical models

Assuming there is an elliptical fracture in an infinite space, the lengths of semi-major and semi-minor axis are a and b respectively, and there is a uniform pressure p_0 acting on the inner boundary of the elliptical fracture. A rectangular coordinate system with the origin at the center of the elliptical fracture, x direction as the semi-major axis and y direction as the semi-minor axis, is built up as shown in Fig. 1.

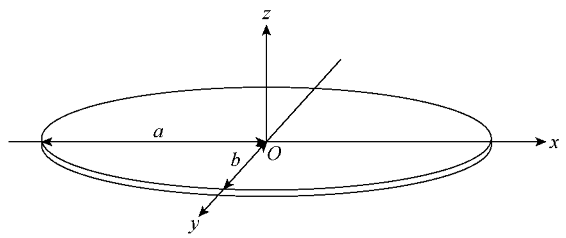

Fig. 1. Physical model of the elliptical fracture.

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The displacements in different directions could be expressed by four harmonic functions, φ_1, φ_2, φ_3 and ψ, in general space problems[13]:

$$\begin{cases} u_x = \varphi_1 + z\dfrac{\partial \psi}{\partial x} \\ u_y = \varphi_2 + z\dfrac{\partial \psi}{\partial y} \\ u_z = \varphi_3 + z\dfrac{\partial \psi}{\partial z} \end{cases} \tag{1}$$

where $\dfrac{\partial \varphi_1}{\partial x} + \dfrac{\partial \varphi_2}{\partial y} + \dfrac{\partial \varphi_3}{\partial z} + (3-4\upsilon)\dfrac{\partial \psi}{\partial z} = 0$

According to the physical model in Fig. 1, the boundary conditions can be expressed as:

$$\begin{cases} \sqrt{x^2+y^2+z^2} \to \infty: \quad \sigma_{ij} \to 0 \\ z=0, (x,y) \in D: \quad \sigma_{zz} = -p_0, \sigma_{xz} = \sigma_{yz} = 0 \\ z=0, (x,y) \notin D: \quad \sigma_{xz} = \sigma_{yz} = 0 \end{cases} \tag{2}$$

In equation (2), σ_{ij} is the stress component, subscripts i and j are the symbols of the summation convention[14], which can be x, y or z respectively.

In this paper, the four harmonic functions can be simplified to one harmonic function f since the elliptical fracture is symmetrical about the plane Oxy, then substitute it into (1):

$$\begin{cases} u_x = (1-2\upsilon)\dfrac{\partial f}{\partial x} + z\dfrac{\partial^2 f}{\partial x \partial z} \\ u_y = (1-2\upsilon)\dfrac{\partial f}{\partial y} + z\dfrac{\partial^2 f}{\partial y \partial z} \\ u_z = -2(1-\upsilon)\dfrac{\partial f}{\partial z} + z\dfrac{\partial^2 f}{\partial z^2} \end{cases} \tag{3}$$

Substituting equation (3) into the displacement-stress relations, the stress components can be expressed by the following equations:

$$\begin{cases} \dfrac{\sigma_{xx}}{2G} = (1-\upsilon)\dfrac{\partial^2 f}{\partial x^2} + \upsilon\dfrac{\partial^2 f}{\partial y^2} - \upsilon\dfrac{\partial^2 f}{\partial z^2} + z\dfrac{\partial^3 f}{\partial x^2 \partial z} \\ \dfrac{\sigma_{yy}}{2G} = (1-\upsilon)\dfrac{\partial^2 f}{\partial y^2} + \upsilon\dfrac{\partial^2 f}{\partial x^2} - \upsilon\dfrac{\partial^2 f}{\partial z^2} + z\dfrac{\partial^3 f}{\partial y^2 \partial z} \\ \dfrac{\sigma_{zz}}{2G} = -\dfrac{\partial^2 f}{\partial z^2} + z\dfrac{\partial^3 f}{\partial z^3} \\ \dfrac{\sigma_{xy}}{2G} = (1-2\upsilon)\dfrac{\partial^2 f}{\partial x \partial y} + z\dfrac{\partial^3 f}{\partial x \partial y \partial z} \\ \dfrac{\sigma_{xz}}{2G} = z\dfrac{\partial^3 f}{\partial x \partial z^2} \\ \dfrac{\sigma_{yz}}{2G} = z\dfrac{\partial^3 f}{\partial y \partial z^2} \end{cases} \tag{4}$$

2. Analytical solution of the model

The definite problem described by equations (2) - (4) can be solved by selecting an appropriate harmonic function f satisfying the boundary conditions. Green et al[15] have found this function in similar hydrodynamic problems:

$$f(x,y,z) = \frac{A}{2}\int_{\xi}^{\infty}\left(\frac{x^2}{a^2+s} + \frac{y^2}{b^2+s} + \frac{z^2}{s} - 1\right)\frac{1}{\sqrt{Q(s)}}\mathrm{d}s \tag{5}$$

where $Q(s) = s(s+a^2)(s+b^2)$

Assuming ξ, ζ, η are the roots of ellipsoid equation $\dfrac{x^2}{a^2+s} + \dfrac{y^2}{b^2+s} + \dfrac{z^2}{s} = 1$, the relationship between them and x, y, z are:

$$\begin{cases} a^2(a^2-b^2)x^2 = (a^2+\xi)(a^2+\eta)(a^2+\zeta) \\ b^2(b^2-a^2)y^2 = (b^2+\xi)(b^2+\eta)(b^2+\zeta) \\ a^2b^2z^2 = \xi\eta\zeta \end{cases} \tag{6}$$

where $-a^2 \le \zeta \le -b^2 \le \eta \le 0 \le \xi \le \infty$

By substituting ξ and s into elliptical transform equation (7), the second derivative of f with respect to z can be obtained (equation (8)). In equations (7) and (8), sn, cn, dn are the Jacobian elliptic sine function, cosine function and amplitude difference function, respectively. E is the incomplete elliptic integral of the second kind, am is the inverse function amplitudes function of the incomplete elliptic integral of the first kind[16].

$$\xi = \frac{a^2 \operatorname{cn}^2 u}{\operatorname{sn}^2 u} = a^2(\operatorname{sn}^{-2} u - 1) \tag{7}$$

$$\frac{\partial^2 f}{\partial z^2} = -\frac{2A}{ab^2}\left[\mathrm{E}(\mathrm{am}u,k) - \frac{\operatorname{cn} u \operatorname{sn} u}{\operatorname{dn} u}\right] + \frac{2A\sqrt{\xi}\left[\xi(a^2b^2-\eta\zeta) - a^2b^2(\eta+\zeta) - \eta\zeta(a^2+b^2)\right]}{a^2b^2\sqrt{a^2+\xi}\sqrt{b^2+\xi}(\xi-\eta)(\xi-\zeta)} \tag{8}$$

Substituting equation (8) into the third equation of (4), in conjunction with the second boundary condition of equation (2), A can be obtained:

$$A = -\frac{ab^2 p_0}{4G\mathrm{E}(\pi/2,k)} \tag{9}$$

where $k = \sqrt{\dfrac{a^2-b^2}{a^2}}$

The expression of elliptic integral is wrong in references [13] and [17], E(u) should be replaced by E(am$u$)[18], otherwise, the final computation result would be wrong. Besides, the description of the value of u at the boundary is inappropriate. In which u would be equal to π/2, when ξ tends to 0. It's a mistake too, since Jacobian elliptic functions are not the inverse functions of the incomplete elliptic integrals of the first kind. Instead, Jacobian elliptic functions are the trigonometric functions of the inverse function amplitudes function am. Therefore, am$u$ would be equal to π/2, when ξ tends to 0.

The explicit expression of f can be achieved by substituting equation (9) into equation (5). The partial derivatives and mixed partial derivatives of f with respect to x, y, z, of different orders can be obtained as complete solutions of stress and displacement. Equations (10) - (24) are modified on the basis of reference [19]. When the amplitude of complete elliptic integral is equal to π/2, the E(am$u$) can be simplified as E(k).