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# Transdimensional inversion of scattered body waves for 1D S-wave velocity structure – Application to the Tengchong volcanic area, Southwestern China

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ABSTRACT

Inversion of receiver functions is commonly used to recover the S-wave velocity structure beneath seismic stations. Traditional approaches are based on deconvolved waveforms, where the horizontal component of P-wave seismograms is deconvolved by the vertical component. Deconvolution of noisy seismograms is a numerically unstable process that needs to be stabilized by regularization parameters. This biases noise statistics, making it difficult to estimate uncertainties in observed receiver functions for Bayesian inference. This study proposes a method to directly invert observed radial waveforms and to better account for data noise in a Bayesian formulation. We illustrate its feasibility with two synthetic tests having different types of noises added to seismograms. Then, a real site application is performed to obtain the 1-D S-wave velocity structure beneath a seismic station located in the Tengchong volcanic area, Southwestern China. Surface wave dispersion measurements spanning periods from 8 to 65 s are jointly inverted with P waveforms. The results show a complex S-wave velocity structure, as two low velocity zones are observed in the crust and uppermost mantle, suggesting the existence of magma chambers, or zones of partial melt. The upper magma chambers may be the heart source that cause the thermal activity on the surface.

## 1. Introduction

Receiver functions (RFs) contain information on seismic structure beneath receivers. They are computed by deconvoluting the vertical components of body wave seismograms from the horizontal component (Vinnik et al., 1977; Langston, 1979; Ammon, 1991; Bostock, 1998). Observed receiver functions can then be inverted to constrain the S-wave velocity (*Vs*) structure beneath the station (Julià et al., 2000; Li and Mashele, 2009; Hammond et al., 2011; Hammond, 2014; Reeves et al., 2015). Various approaches have been implemented, including linearized inversions (Julià et al., 2000; Herrmann and Ammon, 2002; Sosa et al., 2014; Chen et al., 2015, 2016; Li et al., 2016) and non-linear inversions (Sambridge, 1999a; Lawrence and Shearer, 2006; Shen et al., 2016a,b; Kim et al., 2016).

Linearized inversion techniques are based on partial derivatives and the solution model get easily trapped by local minima of the misfit function. Moreover, the final results of linearized inversion strongly depend on the initial model (Julià et al., 2000; Sosa et al., 2014; Wu et al., 2016). Non-linear global optimization techniques, such as genetic algorithm (Shibutani et al., 1996) or simulated annealing (Vinnik et al., 2004), have the ability to efficiently search for a global optimal solution in a highly-dimensional model space. However, these Monte Carlo methods only provide a single best fitting model and fail at representing uncertainty estimates. To overcome this problem, ensemble inference techniques based on a Bayesian formulation of the inverse problem can be used (Mosegaard and Tarantola, 1995; Gallagher et al., 2009; Ball et al., 2014). These ensemble inference techniques provide an ensemble of models sampled from the posterior probability distribution (PPD) using important sampling algorithms, for example the Metropolis-Hasting (M-H) algorithm (Hastings, 1970). The ensemble of models in the solution is used to quantify the credibility of model parameters, providing not only parameters' estimation but also posterior variance and correlation estimates. In recent years, ensemble based Bayesian Monte Carlo techniques have been expanded to the transdimensional case, where the dimension of model space (e.g. number of layers) is unknown and variable (Green, 1995, 2003). After the earliest application of transdimensional Bayesian inversion (TBI) by Malinverno (2002) to solve the inverse problem of DC resistivity sounding, Piana

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Agostinetti and Malinverno (2010) firstly applied this technique to the inversion of RF, where the number of layers was simultaneously inverted with *Vs* values. Transdimensional inversion is becoming popular in the geoscience community, attracting increasing interest due to its flexibility for model parameterization (Malinverno, 2002; Sambridge et al., 2006, 2013; Bodin and Sambridge, 2009; Piana Agostinetti and Malinverno, 2010, Dettmer et al., 2010, 2012, 2015; Gallagher et al., 2011; Petrescu et al., 2016; Zheng et al., 2017).

Differing from linearized and traditional non-linear optimization methods, data uncertainty estimates become vital in Bayesian inference (Gouveia and Scales, 1998). Estimated data errors directly control the form of posterior probability distribution. In a transdimensional formulation, data uncertainty plays an even more critical role, as it directly controls the number of parameters in the solution models, that is the complexity of the model space (Piana Agostinetti and Malinverno, 2010; Bodin et al., 2012). Different sources of errors can contribute to data noise, and each of them has different statistical characteristics. The commonly assumed Gaussian white noise, represented by a diagonal covariance matrix, is not accurate enough to describe uncertainties in observed RFs (Sambridge, 1999a; Piana Agostinetti and Malinverno, 2010; Kolb and Lekić, 2014). Instead, the errors contained in RFs are intrinsically correlated in time due to the deconvolution process within limited frequency band (Piana Agostinetti and Malinverno, 2010). Bodin et al. (2012) expanded TBI to the hierarchical case where the variance and correlation of data errors are all treated as unknowns, assuming a Gaussian correlation function. However, the deconvolution process biases noise statistics in RFs, making it difficult to describe the noise in deconvolved waveforms with simple statistics. By looking at the noise statistics of a large number of realizations for noisy RFs, the covariance matrix of data errors cannot be simply parameterized with a Gaussian correlation function. Dettmer et al. (2012) proposed a procedure to estimate the full data covariance matrix by using an arbitrarily high-order autoregressive error model. Although this procedure is promising at first sight, it assumes an exponential decay for the correlation function, which may not be appropriate for receiver functions.

To address these issues, Bodin et al. (2014) proposed an inversion scheme based on a cross-convolution misfit function, where no deconvolution is needed (Menke and Levin, 2003). This technique has recently been used to constrain the upper mantle structure across North America (Calò et al., 2016), and updated to also include SKS waveforms to constrain anisotropic layering (Bodin et al., 2016). However, this cross-convolution misfit function is not a direct data fit (i.e. the difference between observed and modeled data), but rather a conveniently defined cost function, and it cannot be used to construct a proper likelihood function for Bayesian inference (Frederiksen and Delaney, 2015). More recently, Dettmer et al. (2015) proposed a fully Bayesian direct-seismogram inversion technique for receiver-side structure, where the deconvolution was avoided by treating the source-time function as an unknown in the inversion.

In this study, we propose an alternative likelihood function for Bayesian inversion of scattered body waves. Our approach avoids deconvolving noisy seismograms, as we directly invert the observed radial seismograms. The estimated radial waveform is generated by convolving the synthetic RF (computed for a given earth model) with the observed vertical waveform. In this way, this misfit function represents a direct fit to the observed radial waveform, and can be used to define a likelihood function for Bayesian inference. We note that this misfit function has already been used by Kolb and Lekić (2014) to solve the deconvolution problem and in many other non-Bayesian inversion studies (Kosarev et al., 1984; Farra et al., 1991; Farra and Vinnik, 2000). Although this misfit function is not new, in this work we use it for the first time for Bayesian inversion of scattered body waves.

## 2. Methodology

The proposed approach avoids two common problems in receiver function imaging: (1) the deconvolution of noisy signals, and (2) the estimation of errors in observed RFs. The data vector we are trying to fit is the observed radial component $\mathbf{H}_{obs}(t)$. It can be modeled with the following convolution model:

$$\mathbf{H}_{obs}(t) = \mathbf{h}(t, m)*\mathbf{s}(t) + \mathbf{e}_h(t) \tag{1}$$

where $\mathbf{h}(t, m)$ is the radial impulse response, $s(t)$ is the source-time function and $\mathbf{e}_h(t)$ is random noise with covariance $\mathbf{C_h}$. The radial impulse response can be expressed in terms of the vertical response $\mathbf{h}(t,m) = \mathbf{R}(t,m) * \mathbf{v}(t,m)$, where $\mathbf{R}(t,m)$ is the theoretical receiver function or transfer function. A deconvolution is still needed to compute $\mathbf{R}(t,m)$. However, this is a deconvolution of synthetic seismograms, which is stable as there is no noise involved (Dettmer et al., 2015). We can then write:

$$\mathbf{H}_{obs}(t) = \mathbf{R}(t, m)*\mathbf{v}(t, m)*\mathbf{s}(t) + \mathbf{e}_h(t) \tag{2}$$

This model of the horizontal component still involves the unknown source function $s(t)$. But the term $\mathbf{v}(t,m) * \mathbf{s}(t)$ can be estimated from our observed vertical seismogram:

$$\mathbf{V}_{obs}(t) = \mathbf{v}(t, m)*\mathbf{s}(t) + \mathbf{e}_v(t) \tag{3}$$

where $\mathbf{e}_v(t)$ is a random noise with covariance $\mathbf{C_v}$. We then write:

$$\mathbf{H}_{obs}(t) = \mathbf{R}(t, m)*(\mathbf{V}_{obs}(t) + \mathbf{e_v}(t)) + \mathbf{e_h}(t) \tag{4}$$

$$\mathbf{H}_{obs}(t) = \mathbf{R}(t, m)*\mathbf{V}_{obs}(t) + \mathbf{R}(t, m)*\mathbf{e_v}(t) + \mathbf{e_h}(t) \tag{5}$$

and assuming $\mathbf{e}_v(t)$ and $\mathbf{e}_h(t)$ are normally distributed with zero mean and covariance $\mathbf{C_v}$ and $\mathbf{C_h}$, $\mathbf{H}_{obs}(t)$ can be seen as a vector of random variables with mean $\mathbf{R}(t) * \mathbf{V}_{obs}(t)$ and covariance

$$\mathbf{C_d} = cov(\mathbf{H}_{obs}) = cov(\mathbf{R}*\mathbf{e_v}) + cov(\mathbf{e_h}) = \mathbf{M}\mathbf{C_v}\mathbf{M}^T + \mathbf{C_h} \tag{6}$$

where the matrix $\mathbf{M}$ is defined from the vector $\mathbf{R}$ ($\mathbf{M}[i,j] = \mathbf{R}_{i\text{-}j}$). We have now a noise model for the observed horizontal component, which allows us to write a likelihood probability distribution for $\mathbf{H}_{obs}$ in Eq. (5).

In this way, we can generate synthetic radial waveforms by convolving observed vertical component waveforms $\boldsymbol{V}_{obs}(t)$ with theoretical RFs $\boldsymbol{R}(t,m)$. For a given earth model $\mathbf{m}$, the misfit function $\boldsymbol{\Phi}(t,\mathbf{m})$ is defined as the Mahalanobis distance between the observed and synthetic radial waveforms (Kolb and Lekić, 2014):

$$\Phi(t, m) = (\mathbf{H}_{obs}(t)-\mathbf{R}(t, m)*\mathbf{V}_{obs}(t))^T\mathbf{C}_d^{-1}(\mathbf{H}_{obs}(t)-\mathbf{R}(t, m)*\mathbf{V}_{obs}(t)) \tag{7}$$

This misfit function represents a waveform fit, and hence has a clearer physical meaning than the one proposed by Bodin et al. (2014). The likelihood function of observed radial waveform for a given earth model is then constructed as:

$$P(H|m) = \frac{1}{\sqrt{(2\pi)^n|\mathbf{C_d}|}}\exp\left\{-\frac{1}{2}((\mathbf{H}_{obs}(t)-\mathbf{R}(t, \mathbf{m})*\mathbf{V}_{obs}(t))^T\mathbf{C_d}^{-1}((\mathbf{H}_{obs}(t) -\mathbf{R}(t, \mathbf{m})*\mathbf{V}_{obs}(t))\right\} \tag{8}$$

where $n$ is the number of data points in $\mathbf{H}_{obs}(t)$.

Although we could use the exact form for $\mathbf{C_d}$, in this work assume that $\mathbf{C_v} = \mathbf{0}$, and simply use $\mathbf{C_d} = \mathbf{C_h}$, as done in Kolb and Lekić (2014). Since the amplitude of horizontal component is much smaller than the one from the vertical component, the noise on the vertical component may be negligible. That is, assuming $\mathbf{C_v} = \mathbf{C_h}$, we have $cov(\mathbf{R} * \mathbf{e_v}) \ll cov(\mathbf{e_h})$.

We can then solve our waveform inversion problem using transdimensional hierarchical Bayesian inference, as implemented by Bodin et al. (2012). The number of layers and the parameters for characterizing the covariance matrix are all considered as unknowns. We use the spectral approach of Shibutani et al. (1996) to calculate the theoretical