Available online at www.sciencedirect.com

ScienceDirect

Russian Geology and Geophysics 59 (2018) 105–109

RUSSIAN GEOLOGY AND GEOPHYSICS

www.elsevier.com/locate/rgg

# Diagnostics of the shape and orientation of a rock failure zone based on electrical measurements as an inverse problem of geophysics

D.Yu. Sirota *, V.V. Ivanov

*T.F. Gorbachev Kuzbass State Technical University, ul. Vesennyaya 28, Kemerovo, 650000, Russia*

Received 3 June 2016; receoved in revised form 2 December 2016; accepted 2 August 2017

**Abstract**

Exploration geophysics is concerned with the development of methods and techniques for remote search for mineral deposits, solution of various engineering and geological problems, mining monitoring, and diagnostics of zones of rock bursts and natural and technogenic tectonic earthquakes. In this paper, we consider the diagnostic problem of exploration geophysics related to the determination of the shape and inclination angle of a plane source of natural geoelectric field, which, under certain conditions, simulates a rock failure zone, e.g., during the preparation of rock bursts at the sites of developed mineral deposits. This approach may also be useful in determining the shape and size of ore shoots by electrical measurements on the ground surface. Evaluation of the parameters of a rock failure zone is required in the case of accumulation of multiple fractures having charges of the same sign before a catastrophic failure. This problem is formulated as a Fredholm–Urysohn integral equation of the first kind. The solution of the integral equation is sought using the Tikhonov regularization method of the second order.


*Keywords:* exploration geophysics; diagnostic problem; interpretation of geophysical data; rock burst; integral equation of the first kind; inverse ill-posed problem; Tikhonov regularization method

## Introduction

Exploration geophysics is based on the measurement and subsequent interpretation of various parameters of natural and artificial fields whose changes are determined by the inhomogeneous composition and variable properties of the crust and the natural or man-made processes occurring in it. The problems of exploration geophysics can be classified into diagnostic problems, where there is no need to predict the conditions of rock failure, and prognostic problems, where it is required to predict, e.g., the location of a failure zone and the strength of a possible seismic event.

The problems of determining the source parameters of a tectonic earthquake or rock burst take a special place among the problems of determining the shape and size of a potential field source. As has been shown previously (Ivanov et al., 2013), due to the accumulation of multiple cracks whose tips have charges of the same sign at the final stage of preparation of a rock burst, the electric field perturbations in rocks on the ground and in the atmosphere can reach breakdown values, causing atmospheric glow and breakdown of electrical cables in the ground. In this case, the source zone is concentrated in a narrow region of a tectonic fault which can be approximated by a substantially plane ellipsoid (Ivanov et al., 2013). Thus, electromagnetic field perturbations at the final stages of preparation of an earthquake can be used to determine its geometrical parameters.

From the viewpoint of applied mathematics, all problems of interpretation of measured data are inverse and ill-posed (Bakushinsky et al., 2011; Kabanikhin, 2009; Neto and Neto, 2013; Tarantola, 2005; Tikhonov and Glasko, 1964, 1965; Zhdanov, 2007). Currently, the main method for solving such problems is to use the Tikhonov regularizing functional (Tikhonov and Glasko, 1964, 1965), whose minimum corresponds to the solution of the inverse problem. In the present paper, we discuss the diagnostic problem of determining the shape and inclination angle of a plane potential field source from measurements of the potential or intensity on the ground surface, which reduces to solving the Fredholm–Urysohn integral equation of the first kind. This problem has been solved previously (Tikhonov and Glasko, 1964; Zhdanov, 2007; Zhdanov et al., 2011), but in a simpler formulation.

* Corresponding author.
*E-mail address:* dmsirota@yandex.ru (D.Yu. Sirota)

1068-7971/$ - see front matter 
http://dx.doi.org/10.1016/j.rgg.2018.01.008

## Formulation and solution of the forward problem

Suppose that an electric field (in what follows, we consider only natural electric fields (NEF), but the proposed method is applicable to any potential fields due to the versatility of the mathematical model used) is generated by a plane source of arbitrary shape $S_p$ which has an inclination angle $\gamma \in [0°; 70°]$ and is located at a certain depth $z_M = H$ (without crossing the boundaries of the first and second layers) in the lower layer of a three-layer homogeneous and isotropic host medium (Fig. 1).

The magnitude of the field potential of this source at an arbitrary measurement point $M$ on the ground surface will be determined in polar coordinates according to the general formula

$$U^M = \int_{S_p} u^M \, dS_p. \quad (1)$$

where $S_p$ is the plane current source and the function $u^M$ is given by the well-known formula of the potential of a point current source (Bursian, 1972)

$$u^M = C \int_0^{\infty} \frac{J_0(m \cdot D) \exp(-m \cdot Z_M)}{1 + W \exp(-m \cdot h)} dm, \quad (2)$$

where $D = \sqrt{(X_M - X_p)^2 + (y_M - y_p)^2}$ is the distance from the point $P$ of the integration region to the field measurement point $M$ (m); $C$ is a coefficient that characterizes the electric current intensity of the source; $J_0(m \cdot D)$ is a Bessel function of zero order; $h$ is the thickness of the second layer (m); $W = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$ is the reflection coefficient of the second layer; $\rho_{1,2}$ is the electrical resistivity of the first and second layers (Ohm·m); $X_M = x_M \cos\gamma + z_M \sin\gamma$ and $Z_M = -x_M \sin\gamma + z_M \cos\gamma$ are the formulas of transformation of the coordinates of the measurement point from the $Oxz$ system to the $OXZ$ system; $\gamma$ is the angle of rotation of the $Oxz$ system; and $m$ is an integration variable.

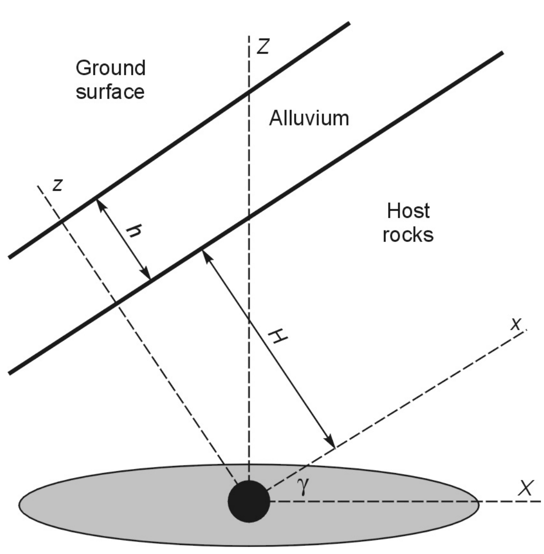


Fig. 1. Schematic of the host medium and plane field source.

Here and below, it is assumed that the depth of the NEF source $z_M = H$ is known and only the results of measurements along the $Ox$ axis are used; i.e., $y_M = 0$.

To integrate over an arbitrary region with a closed boundary, we pass from the Cartesian coordinate system to the polar coordinate system according to the following standard formulas: $X_p = r\cos\varphi$ and $Y_p = r\sin\varphi$, where $\varphi \in [0; 2\pi]$ and $r \in [0; \rho(\varphi)]$, with the function $\rho(\varphi)$ defining the circuit of the plane current source.

Next we transform expression (2) to the product of the dimensional and dimensionless factors, making the changes of variables: in the outer integral, $w \cdot h^{-1} = \overline{w}$, where $w$ denotes all parameters expressed in units of [m], and in the inner integral, $m \cdot h = \overline{m}$. Then integral (1) becomes

$$U^M = h \cdot C \int_{\overline{S}_p} \overline{r} \left[ \int_0^{\infty} \frac{J_0(\overline{m} \cdot \overline{D}) \exp(-\overline{m} \cdot \overline{Z}_M)}{1 + W \exp(-\overline{m})} d\overline{m} \right] d\overline{r} \, d\varphi, \quad (3)$$

where $\overline{D} = \sqrt{(\overline{X}_M - \overline{r}\cos\varphi)^2 + \overline{y}_M - \overline{r}\sin\varphi)^2}$.

To calculate the inner improper integral, we approximate the integrand fraction $\frac{1}{1 + W\exp(-\overline{m})} = \sum_{k=1}^{11} q(k) \exp[-\overline{m}(k-1)]$, where the coefficients $q(k)$ are determined by the least squares method.

Applying the Weber–Lipschitz integral yields the following formula for calculating the magnitude of the potential at an arbitrary point $M$ on the ground surface:

$$U^M = C \cdot h \int_0^{2\pi} \left[ \sum_{k=1}^{11} q(k) \int_0^{\overline{\rho}(\varphi)} \frac{\overline{r} \, d\overline{r}}{\sqrt{R}} \right] d\varphi. \quad (4)$$

Here $R = \overline{r}^2 + B \cdot \overline{r} + A$, $A = \overline{X}_M^2 + (\overline{Z}_M + k - 1)^2$, $B = -2\overline{X}_M \cos\varphi$, and the inner integral in (4) is equal to

$$\int_0^{\overline{\rho}(\varphi)} \frac{\overline{r} \, d\overline{r}}{\sqrt{R}} = \left( \sqrt{R} - \frac{B}{2} \ln \left| \overline{r} + \frac{B}{2} + \sqrt{R} \right| \right) \Bigg|_0^{\overline{\rho}(\varphi)}$$

$$= F(\overline{X}_M, \overline{Z}_M, \overline{\rho}(\varphi), \gamma).$$

To determine the vertical intensity component of the NEF, we calculate the derivative of the function $U_M$ with respect to the variable $\overline{z}_M$:

$$E^M = C \cdot h \int_0^{2\pi} \left[ \sum_{k=1}^{11} q(k) \int_0^{\overline{\rho}(\varphi)} \frac{\overline{r} \cdot k d\overline{r}}{\sqrt{R^3}} \right] d\varphi, \quad (5)$$

where $k = \overline{r}\sin\gamma\cos\varphi + (\overline{Z}_M + k - 1)\cos\gamma$.

The inner integral in (5) is also quite easy to calculate, but calculations have shown that the resulting formula is not well suited for numerical implementation due to its complexity and