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Mixing laws and causality in high frequency induction log applications

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Abstract

High frequency electromagnetic technologies for subsurface formation evaluation provide high spatial resolution and new opportunities for petrophysical interpretation of data. Dispersion of rock properties and up-scaling from pore to reservoir scale (homogenization) represent the two most challenging problems. In electrodynamics of porous media, various mixing and dispersion laws are used to homogenize rock properties and describe their frequency behavior. Mixing laws and dispersion have a close link to the fundamental physical principle of causality and therefore cannot be introduced arbitrarily. For any mixing/dispersion law, we need to prove that causality holds. For testing causality, we use *Titchmarsh's theorem* and, particularly, one of its modifications—*Kramers–Kronig relations*. Causality is discussed for *Debye, Cole–Cole, Havriliak–Negami, and CRIM* models. Dispersion is closely related to wave propagation. Evaluation of phase and group velocities shed new light on the physics of phase and amplitude measurements in lossy media. We evaluated various definitions of both velocities and their dependence on spatial spectra or, in other words, on the arrangement of transmitting and receiving elements. To illustrate theoretical findings, we use dielectric logging as an exemplary technology. Usually, in modern dielectric tools, amplitude and phase data are acquired, for various frequencies and sensor positions. The measured phase is discontinuous at high frequencies and requires detection of discontinuity as well as unwrapping. Remarkably, one can determine formation attenuation and loss angle based on multifrequency/multisensor amplitude data and transform them into dielectric permittivity, resistivity, and true continuous phase. Transformations of exemplary tool data used in this paper are suitable for a conceptual study and are specific for a uniform formation. We intentionally do not address the accuracy of measurements and propagation of errors in the inversion process, since they are tool- and processing-specific. Different tools require joint analysis of all available data and special noise reduction techniques associated with the structure of the acquisition system.

Keywords: electromagnetics; dielectric; logging; dispersion; mixing laws; causality

1. Introduction

High frequency electromagnetic technologies for subsurface formation evaluation provide high spatial resolution and new opportunities for petrophysical interpretation of data. Dispersion of rock properties and up-scaling from pore to reservoir scale (homogenization) represent the two most challenging problems.

In electrodynamics of porous media, various mixing and dispersion laws are used to homogenize rock properties and describe their frequency behavior. Mixing laws and dispersion have a close link to the fundamental physical principle of causality and therefore cannot be introduced arbitrarily. For any mixing/dispersion law, we need to prove that causality holds. (Alu et al., 2011) discuss an example of causality violation in Maxwell–Garnett mixing law. For testing causality, we use *Titchmarsh's theorem* (Nordebo, 2013; Titchmarsh, 1926; Toll, 1956) and, particularly, one of its modifications—*Kramers–Kronig relations*. Causality is discussed for *Debye, Cole–Cole, Havriliak–Negami, and CRIM* models.

Dispersion is closely related to wave propagation. Evaluation of phase and group velocities shed new light on the physics of phase and amplitude measurements in lossy media. We evaluated various definitions of both velocities and their dependence on spatial spectra or, in other words, on the arrangement of transmitting and receiving elements.

To illustrate theoretical findings, we use dielectric logging as an exemplary technology. Usually, in modern dielectric tools, amplitude and phase data are acquired, for various frequencies and sensor positions. The measured phase is discontinuous at high frequencies and requires detection of discontinuity as well as unwrapping (Abbas, 2005). Remarkably, one can determine formation attenuation and loss angle based on multifrequency/multisensor amplitude data and trans-

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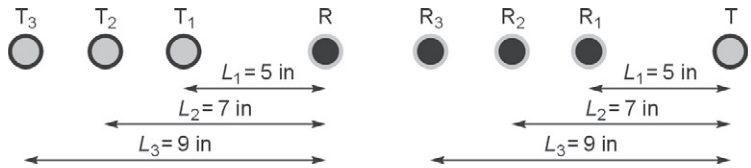

Fig. 1. Generic tool schematics.

form them into dielectric permittivity, resistivity and true continuous phase.

Transformations of exemplary tool data used in this paper are suitable for a conceptual study and are specific for a uniform formation. We intentionally do not address accuracy of measurements and propagation of errors in the inversion process since they are tool and processing specific. Real tools require joint analysis of all available data and special noise reduction techniques associated with the structure of the acquisition system.

We start the paper with description of generic dielectric tool (Section 2) that is used in following discussion of phase and group velocities, spectra, mixing laws, and causality (Sections 3–5).

2. Generic tool

We consider a generic tool schematically shown in Fig. 1. On the left, three transmitters (magnetic dipoles), T_1, T_2, and T_3, generate EM field measured by the sensor, R. A reciprocal configuration is shown on the right.

2.1. Measurements and useful field transformations

Let us consider a signal generated by a single transmitter in a single receiver. The normalized magnetic field, $h_z = H_z/(M/2\pi L^3$, may be expressed in the following way (Kaufman and Keller, 1989):

$$h_z = e^{-kL}(1 + kL)\,e^{i\omega t}, \tag{1}$$

$$k^2 = -i\omega\mu(\sigma - i\omega\varepsilon) = -\omega^2\mu\varepsilon - i\omega\mu\sigma. \tag{2}$$

Here, M, L—transmitter moment (A·m^2) and spacing (m), respectively; $\omega = 2\pi f$, where f is frequency (Hz); t—time; σ—formation conductivity (S/m); $\varepsilon = \varepsilon^* \cdot \varepsilon_0$—formation dielectric permittivity (F/m); $\mu = \mu^* \cdot \mu_0$—formation magnetic permeability (H/m); $\varepsilon_0 = 10^{-9}/(36\pi)$ F/m—dielectric permittivity of free space; $\mu_0 = 4\pi \times 10^{-7}$ H/m—magnetic permeability of free space; ε^*, μ^*—permittivity and permeability relative to free space.

It follows from Eq. (2) that the complex number k^2 may belong only to the third quarter of a complex plane. Let us consider the following representation of k:

$$k = |k|\,e^{i\varphi_k} = |k|\left(\cos(\varphi_k) + i\sin(\varphi_k)\right), \tag{3}$$

$$|k| = \sqrt{\omega\mu\sqrt{(\omega\varepsilon)^2 + \sigma^2}}, \tag{4}$$

$$\varphi_k = \frac{1}{2}\,\mathrm{atan}\,2\,(-\omega\varepsilon, -\sigma), \tag{5}$$

$$-\pi/2 \le \varphi_k \le -\pi/4. \tag{6}$$

Here, atan2(x, y) means an argument of a complex number with real and imaginary parts equal x and y, respectively. Angle φ_k closely relates to the loss angle δ in formation: $\tan\delta = \tan(2\varphi_k) = \sigma/\omega\varepsilon$.

We will need the following representation of the function $(1 + kL)$ in Eq. (1):

$$(1 + kL) = (1 + |k|L\cdot\cos(\varphi_k)) + i\,(|k|L\cdot\sin(\varphi_k)) = |1 + kL|\,e^{i\Psi}, \tag{7}$$

$$|1 + kL| = A(L) = \sqrt{(1 + |k|L\cdot\cos(\varphi_k))^2 + (|k|L\cdot\sin(\varphi_k))^2}$$
$$= \sqrt{1 + 2|k|L\cdot\cos(\varphi_k) + (|k|L)^2}\ , \tag{8}$$

$$\psi = \arctan\left(\frac{|k|L\cdot\sin(\varphi_k)}{1 + |k|L\cdot\cos(\varphi_k)}\right),\ \varphi_k \le \psi \le 0,\ |k\,L| \in (0, \infty). \tag{9}$$

Equations (1)–(9) result in the following expression for the normalized magnetic field, h_z:

$$h_z = A(L)\,e^{-\alpha - i\Phi},\ \alpha = |k|L\cos(\varphi_k),$$
$$\Phi = |k|L\sin(\varphi_k) - \psi + \omega t. \tag{10}$$

Assuming $L_3 - L_2 = L_2 - L_1$ we introduce the following transformations of three magnetic fields produced by three transmitters in the receiver R:

$$D_2 = \frac{|h_z(L_3)|}{|h_z(L_2)|} = \frac{A(L_3)}{A(L_2)}\,e^{-|k|(L_3 - L_2)\cos(\varphi_k)}, \tag{11}$$

$$D_3 = \frac{|h_z(L_3)|}{|h_z(L_2)|}\frac{|h_z(L_1)|}{|h_z(L_2)|} - \frac{L_3 L_1}{(L_2)^2} = \frac{A(L_3)\,A(L_1)}{(A(L_2))^2} - \frac{L_3 L_1}{(L_2)^2}. \tag{12}$$

Exemplary transformations (11) and (12) are useful for determining formation parameters, σ and ε. Other transformations may be considered as well. Please notice that $D_3 \to 0$ when $\omega \to \infty$. It provides increased sensitivity to formation parameters at high frequencies though requires improved accuracy of measurements.

2.2. Exemplary model and inversion

Given field transformations, D_2 and D_3, at a certain frequency, f, we can determine formation parameters, σ and ε. To illustrate the method, we selected the following model:

σ = 1.08 S/m; ε^* = 55.62; f = 293,311,000 Hz.