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Fast computation of MT curves for a horizontally layered earth with laterally inhomogeneous conductivity perturbations

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Abstract

A new algorithm is proposed to compute magnetotelluric (MT) curves for a horizontally layered earth with laterally inhomogeneous conductivity. It is fast and ensures correction of induced eddy currents and galvanic distortions of MT curves produced by 3D inhomogeneities. The computation time is short (~1 min) due to the use of the perturbation method for solving Maxwell's equations. The suggested algorithm has a better performance than the more costly classical Trefftz method but has an applicability limitation.

Keywords: magnetotelluric soundings of 3D inhomogeneous earth; static shift; induction distortion; fast algorithm; MT curves; electrical conductivity

Introduction

Magnetotelluic (MT) responses of a complex earth with 3D inhomogeneities are commonly distorted by induced eddy currents and galvanic distortion (i.e., bear a static shift) and thus differ from theoretical normal curves of depth-dependent electrical conductivity for a horizontally layered earth under the station. If remaining uncorrected, the static shift leads to misinterpretation of the collected MT profiles and to a wrong resistivity model. Specifically, distortions produced by shallow structures may be misinterpreted as deep conductors. Much research has aimed at improving the reliability and performance of MT data, and many attempts have been undertaken to account for the earth complexity and to correct static shifts (Berezina et al., 2013; Ivanov and Pushkarev, 2010; Pellerin and Hohmann, 1990; Sasaki, 2004; Zinger and Fainberg, 2005).

Efficient ways for static shift remediation which would require low computation costs and little change to the existing acquisition tools are urgent for the practice of MT soundings. In a previous study, we (Plotkin and Gubin, 2015) suggested an algorithm for static shift correction based on the Trefftz method. In this method, the earth is presented as an assemblage of constant-conductivity blocks, and the modeling domain consists of several laterally inhomogeneous layers of equal blocks lying over a homogeneous subsurface. Inversion seeks conductivities within the blocks of each inhomogeneous layer, thicknesses of these layers, and conductivity below them. The inversion algorithm was tested on synthetic data and was applied to interpret distorted MT curves from fault zones of Gorny Altai (Plotkin et al., 2017). With this approach, inversion can be run on an ordinary desktop PC, at a computation time and to an approximation degree depending on the number of blocks. In the cited studies, the model consisted of three layers, each with 25 blocks (five blocks along the *OX* axis), and altogether 79 exponential parameters were sought.

This study presents another numerical model which provides much faster computation for times greater numbers of blocks and layers than the previous algorithm. Its main limitation is in the applicability of the perturbation method used for solving the Maxwell equation.

Numerical model

Let the earth have the electrical conductivity $\sigma(x, y, z) = \sigma_0(z) + \sigma'(x, y, z)$, and $\sigma_0(z)$ be a piecewise constant function (normal depth-dependent conductivity). Maxwell's equations are solved with the method of perturbation, at $\sigma_0(z) \gg \sigma'(x, y, z)$, as the series

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$$\mathrm{rot}\left(\mathbf{E}^{(0)}+\mathbf{E}^{(1)}+\ldots\right)=-i\omega\mu_0\left(\mathbf{H}^{(0)}+\mathbf{H}^{(1)}+\ldots\right),$$
$$\mathrm{rot}\left(\mathbf{H}^{(0)}+\mathbf{H}^{(1)}+\ldots\right)=(\sigma_0(z)+\sigma')\left(\mathbf{E}^{(0)}+\mathbf{E}^{(1)}+\ldots\right). \quad (1)$$

In the zero approximation, for vertical plane-wave incidence,

$$\Delta\mathbf{E}^{(0)}-k_0^2\mathbf{E}^{(0)}=0,\quad k_0^2=i\omega\mu_0\sigma_0(z). \quad (2)$$

In the first approximation, at zero field divergence $\mathrm{div}\left[(\sigma_0(z)+\sigma')\left(\mathbf{E}^{(0)}+\mathbf{E}^{(1)}+\ldots\right)\right]=0$, we obtain

$$\mathrm{div}\left(\mathbf{E}^{(1)}\right)=-\frac{1}{\sigma_0(z)}\mathbf{E}^{(0)}\,\mathrm{grad}\,\sigma',$$
$$\Delta\mathbf{E}^{(1)}-k_0^2\mathbf{E}^{(1)}=i\omega\mu_0\sigma'\mathbf{E}^{(0)}-\frac{1}{\sigma_0(z)}\mathbf{E}^{(0)}\,\mathrm{grad}\,\sigma'. \quad (3)$$

The second equation for the first-approximation field (3) differs from the zero-approximation case (2) only in the presence of a right-hand side as a two-dimensional Fourier series with known coefficients. Note that $\mathrm{div}\left(\mathbf{E}^{(1)}\right)\neq 0$ in the first approximation, which means the TM mode excitation ($E_z\neq 0$) and additional distortions in MT curves (note that the derivation in (Plotkin, 2013) did not include the last term in the right-hand side of (3)).

It is convenient to present Maxwell's equations (1), for both zero- (2) and first-approximation (3) solutions, as a system of first-order equations (Alexandrov, 2001) for space-time field harmonics $\sim\exp(i\omega t+ik_xx+ik_yy)$ in the matrix form:

$$\frac{d\mathbf{X}}{dz}=\mathbf{AX},\quad \mathbf{X}=\left(H_x,H_y,E_x,E_y\right)^T, \quad (4)$$

where $\mathbf{X}$ is the vector of horizontal field components, T denotes transposition, and the matrix $\mathbf{A}$ is

$$\mathbf{A}=\begin{pmatrix} 0 & 0 & -\frac{k_xk_y}{i\omega\mu_0} & \frac{k_x^2}{i\omega\mu_0}+\sigma_0(z) \\ 0 & 0 & -\frac{k_x^2}{i\omega\mu_0}-\sigma_0(z) & \frac{k_xk_y}{i\omega\mu_0} \\ \frac{k_xk_y}{\sigma_0(z)} & -i\omega\mu_0-\frac{k_x^2}{\sigma_0(z)} & 0 & 0 \\ i\omega\mu_0+\frac{k_y^2}{\sigma_0(z)} & -\frac{k_xk_y}{\sigma_0(z)} & 0 & 0 \end{pmatrix}. \quad (5)$$

Let the axis OZ be directed depthward (inward the horizontally layered earth), and the depth-dependent conductivity $\sigma_0(z)$ be expressed via the earth's parameters σ_n, d_n, and $k_{zn}=\sqrt{k_x^2+k_y^2+i\omega\mu_0\sigma_n}$, $n=1,\ldots,N$, $d_N\to\infty$. Then, the transposition of the horizontal field components across a homogeneous layer of the thickness d_n, according to (4), is given by

$$\mathbf{X}_{n+1}=e^{\mathbf{A}_nd_n}\mathbf{X}_n,\quad \mathbf{A}_n=\mathbf{A}(\sigma_n). \quad (6)$$

The matrix exponents are commonly calculated (see below) using *MatLab*. If the matrix $\mathbf{A}_n$ can be reduced to the diagonal form (this condition fulfills in all numerical calculations), the exponents can be found as

$$e^{\mathbf{A}_nd_n}=\mathbf{CSC}^{-1}, \quad (7)$$

where $\mathbf{C}$ is the matrix with eigen vectors $\mathbf{A}_n$ in its columns, $\mathbf{S}$ is the diagonal matrix with the respective exponents $\exp(k_{zn}d_n)$ on the principal diagonal, and k_{zn} are the eigen values of the matrix $\mathbf{A}_n$. The extrapolation of the horizontal field components $\mathbf{X}_0$ from the earth surface depthward to the interface with the underlying homogeneous subsurface is described by time-order product of matrix exponents for all intermediate layers. For the homogeneous subsurface, one has to ensure the absence of the solutions $\mathbf{X}_N^+$ that grow at $z\to\infty$, which means that

$$\mathbf{X}_N^+=\mathbf{C}\tilde{\mathbf{S}}\mathbf{C}^{-1}e^{\mathbf{A}_{n-1}d_{n-1}}\ldots e^{\mathbf{A}_1d_1}\mathbf{X}_0=\mathbf{DX}_0=0,$$
$$\mathbf{D}=\mathbf{C}\tilde{\mathbf{S}}\mathbf{C}^{-1}e^{\mathbf{A}_{n-1}d_{n-1}}\ldots e^{\mathbf{A}_1d_1}, \quad (8)$$

where the diagonal matrix $\tilde{\mathbf{S}}$ (compare with (7)) has 1 and 0 instead of the increasing and decreasing exponents, respectively. At $\mathbf{DX}_0=0$, the impedance tensor is calculated as (Alexandrov, 2001):

$$\mathbf{Z}=\begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy}\end{pmatrix}=-\begin{pmatrix} D_{13} & D_{14} \\ D_{23} & D_{24}\end{pmatrix}^{-1}\begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22}\end{pmatrix}$$
$$=-\begin{pmatrix} D_{33} & D_{34} \\ D_{43} & D_{44}\end{pmatrix}^{-1}\begin{pmatrix} D_{31} & D_{32} \\ D_{41} & D_{42}\end{pmatrix}, \quad (9)$$

where D_{ik} are the elements of the matrix $\mathbf{D}$. This procedure is valid for any spatial harmonic, including the zero-approximation wave with $k_x=k_y=0$.

If the primary source of the MT field fits the normal plane incidence (which fulfills in middle latitudes), the spatial field harmonics recorded on the surface are obviously of deep origin. These harmonics in a layered earth are associated with the conductivity perturbations $\sigma'(x,y,z)$ and, hence, with the right-hand sides of the first-approximation equations (3). The right-hand sides are determined by both the magnitude of the zero-approximation field in the layer containing the perturbations $\sigma'(x,y,z)$ and by the amplitudes of their spatial harmonics.

Fields of laterally inhomogeneous thin layers

Let some layer at the depth z' enclose a thin conducting layer with $\sigma'(x,y,z)=\Sigma(x,y)\delta(z-z')$, where $\Sigma(x,y)$ is its total longitudinal conductance and $\delta(z)$ is the delta function. The first-approximation equation for this case is

$$\Delta E_{x,y}^{(1)}-k_0^2E_{x,y}^{(1)}=F_{x,y}\delta(z-z'),$$
$$F_x=E_x^{(0)}(z')\left(i\omega\mu_0\Sigma-\frac{1}{\sigma_0(z')}\frac{\partial^2\Sigma}{\partial x^2}\right)-E_y^{(0)}(z')\frac{1}{\sigma_0(z')}\frac{\partial^2\Sigma}{\partial x\partial y}, \quad (10)$$
$$F_y=-E_x^{(0)}(z')\frac{1}{\sigma_0(z')}\frac{\partial^2\Sigma}{\partial x\partial y}+E_y^{(0)}(z')\left(i\omega\mu_0\Sigma-\frac{1}{\sigma_0(z')}\frac{\partial^2\Sigma}{\partial y^2}\right).$$