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Modeling of frequency-domain elastic-wave equation with a general optimal scheme

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ABSTRACT

Frequency-domain numerical modeling is an important foundation of frequency-domain full waveform inversion.However, the conventional 9-point scheme for frequency-domain 2D elastic-wave equation requires lots of sampling grid points but with limited precision.In this paper, we develop a general optimal 9-point scheme for frequency-domain 2D elastic-wave equation to reduce sampling grid points and improve the accuracy.The numerical dispersion analysis shows that this scheme can effectively reduce the grid point of per wavelength. The new scheme can be applied to equal and unequal spatial sampling intervals, which is convenient for their applications in practice.Accuracy analysis demonstrates that the results of the optimal nine-point scheme are in better agreement with the analytic solutions relative to the conventional 9-point scheme.We deduce the perfectly matched layer (PML) absorbing boundary condition to eliminate the artificial boundary influence.Two numerical examples are used to demonstrate the effectiveness of the general optimal scheme for frequency-domainelastic-wave equation.

1. Introduction

In recent years, full waveform inversion (FWI) has played one of the most important roles in the research of exploration geophysics.FWI can be described as a subsurface parameters optimization process using all types of waves information from seismograms (Virieux and Operto, 2009). FWI has made important progress in both time domain (Boonyasiriwat etal., 2009; Bunks etal., 1995; Gauthier etal., 1986; Tarantola, 1984) and frequency domain (Pratt et al., 1998; Pratt and Worthington, 1990; Shin, 2006).

As we all know, forward modeling is the basis of the inversion, and it can largely affect the quality of inversion results.Frequency-domain forward modeling was pioneered by Lysmer and Drake (1972) using finite-element method to calculate the wave propagation in the earth. This method was developed further by Marfurt (1984) and Marfurt and Shin (1989). Pratt and Worthington (1990) and Pratt (1990a, 1990b)applied finite-difference forward modeling in the frequency-domain to the inversion and seismic imaging.Compared with conventional time domain forwarding modeling, finite-difference forward modeling in the frequency-domain has its own advantages.First, it does not have any stability limitation problems (Marfurt, 1984) because of using implicit finite-difference operator.Second, when the impedance matrix is decomposed by direct solver, it is suitable for multishot parallel computation.Moreover, it is convenient to choose a few frequency components for forward modeling and inversion (Pratt and Worthington, 1990). However, frequency-domainfinite-difference modeling has one obvious disadvantage that it requires more grid points per wavelength than simulating in time-domain for the same accuracy.

By combining a rotated coordinate and the technique of mass acceleration term,Jo etal. (1996) developed an optimal 9-point scheme for the scalar wave equation.This scheme successfully reduced the grid points per wavelength to less than 4.Using the same method, Shin and Sohn (1998) developed an optimal 25-point scheme for the scalar wave equation and reduced the grid points per wavelength to 2.5. Hustedt etal. (2004) and Operto etal. (2007) generalized the rotated coordinate method to variable density case and 3D case, respectively. Based on Jo etal.'s (1996) method and staggered-grid technique, Štekl and Pratt (1998) proposed an optimal 9-point scheme for frequency-domainelastic-wave modeling.Min etal. (2000) developed an optimal weighted-averaging25-point scheme for the elastic wave equation. Min etal.'s (2000) method reduced the grid points per wavelength to 2.7, achieving considerable success.However, based on the rotated coordinate, those optimal schemes require the equal spatial sampling intervals, which greatly affected their applications in practice.To overcome this limitation, Chen (2012) presented an optimal 9-point scheme

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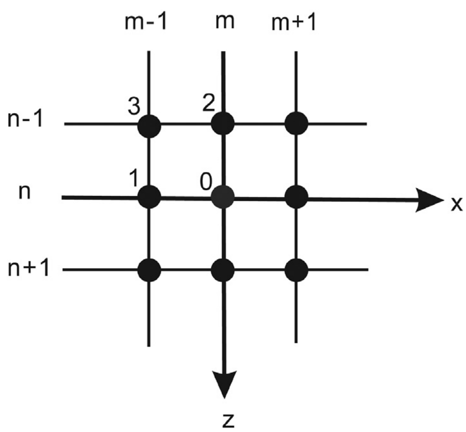

Fig.1. The stencil of the optimal nine-point scheme.

based on average-derivate method (AMD) for acoustic wave equation. Considering the precision and calculation, Tang etal. (2015) proposed an optimal AMD17-point scheme for acoustic wave equation.Chen (2014) directly extended average-derivate method to 3D case and proposed an optimal AMD27-point scheme for 3D acoustic wave equation. Chen and Cao (2016) introduced average-derivate method to frequency-domainelastic-wave modeling. Fan etal. (2017) presented a general optimal method for the scalar wave equation, which had quite good results.

In this paper, we develop a general optimal 9-point method for frequency-domainelastic-wave equation.Our method use 9 grid points to approximate finite-difference operators for 2D frequency-domainelastic-wave equation, as does the method proposed by Fan etal. (2017) for the scalar wave equation.This new general optimal 9-point method has very high accuracy and could be applied to the unequal spatial sampling intervals.

The outline of the paper is as follows.We begin presenting the general optimal scheme for frequency-domainelastic-wave equation.Then, we show the optimization of the coefficients and the numerical dispersion analysis.This is followed by deducing the perfectly matched layer (PML) absorbing boundary condition and accuracy analysis.Finally, Numerical examples are presented to demonstrate the theoretical analysis.

2. Theory

2.1. A general optimal method

In a 2D Cartesian coordinate system, the frequency-domain elastic wave equations in a homogeneous medium are

$$\rho\omega^2 u + (\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial z^2} + (\lambda+\mu)\frac{\partial^2 v}{\partial x \partial z} = 0, \tag{1}$$

Table1
The Optimization coefficients for different Poisson's ratio σ with $r = \Delta x/\Delta z = 1$.

σ	0.1	0.2	0.3	0.4	0.45
b_0	0.6125	0.6047	0.5935	0.5902	0.6027
b_1	0.0957	0.1007	0.1064	0.1092	0.1044
b_2	0.0957	0.1007	0.1064	0.1092	0.1044
b_3	0.00046	−0.0019	−0.0048	−0.0067	−0.0050
c_0	−1.3427	−1.2921	−1.2292	−1.1467	−1.0893
c_1	0.6760	0.6499	0.6173	0.5745	0.5451
c_2	−0.3343	−0.3590	−0.3895	−0.4288	−0.4562
c_3	0.1648	0.1776	0.1934	0.2138	0.2279
d_0	−1.3472	−1.2921	−1.2293	−1.1467	−1.0893
d_1	−0.3343	−0.3590	−0.3894	−0.4288	−0.4562
d_2	0.6760	0.6499	0.6173	0.5745	0.5451
d_3	0.1648	0.1776	0.1934	0.2138	0.2279

Table2
The Optimization coefficients for different Poisson's ratio σ with $r = \Delta x/\Delta z = 1.25$.

σ	0.1	0.2	0.3	0.4	0.45
b_0	0.6116	0.5988	0.5851	0.5817	0.5985
b_1	0.0991	0.1051	0.1123	0.1158	0.1094
b_2	0.0983	0.1044	0.1113	0.1139	0.1067
b_3	−0.0016	−0.0045	−0.0081	−0.0103	−0.0077
c_0	−1.4486	−1.3802	−1.2978	−1.1915	−1.1174
c_1	0.7306	0.6952	0.6523	0.5971	0.5591
c_2	−0.2799	−0.3137	−0.3541	−0.4057	−0.4418
c_3	0.1368	0.1543	0.1753	0.2022	0.2207
d_0	−1.2816	−1.2418	−1.1907	−1.1223	−1.0746
d_1	−0.3646	−0.3840	−0.4086	−0.4410	−0.4636
d_2	0.6433	0.6230	0.5968	0.5618	0.5375
d_3	0.1811	0.1910	0.2036	0.2202	0.2317

$$\rho\omega^2 v + (\lambda+2\mu)\frac{\partial^2 v}{\partial z^2} + \mu\frac{\partial^2 v}{\partial x^2} + (\lambda+\mu)\frac{\partial^2 u}{\partial x \partial z} = 0, \tag{2}$$

where u and v are the horizontal and vertical displacements, respectively, ρ is the density, ω is the circular frequency, λ and μ are the Lamé parameters.

Set $u_{m,n} = u(m\Delta x, n\Delta z)$ and $v_{m,n} = v(m\Delta x, n\Delta z)$, where Δx and Δz are the finite difference grid spacing in x-direction and z-direction, respectively.The conventional nine-point scheme for Eq.(1) and Eq.(2) are

$$\begin{aligned}&\frac{\lambda+2\mu}{\Delta x^2}\left(u_{m-1,n}+u_{m+1,n}-2u_{m,n}\right)+\frac{\mu}{\Delta z^2}\left(u_{m-1,n}+u_{m+1,n}-2u_{m,n}\right)\\&\frac{\lambda+\mu}{4\Delta x\Delta z}\left(v_{m+1,n+1}-v_{m+1,n-1}-v_{m-1,n+1}+v_{m-1,n-1}\right)=0\end{aligned}. \tag{3}$$

$$\begin{aligned}&\frac{\lambda+2\mu}{\Delta x^2}\left(v_{m-1,n}+v_{m+1,n}-2v_{m,n}\right)+\frac{\mu}{\Delta z^2}\left(v_{m-1,n}+v_{m+1,n}-2v_{m,n}\right)\\&\frac{\lambda+\mu}{4\Delta x\Delta z}\left(u_{m+1,n+1}-u_{m+1,n-1}-u_{m-1,n+1}+u_{m-1,n-1}\right)=0\end{aligned} \tag{4}$$

Now, we introduce a new optimal scheme for above elastic wave equations.According to the Fan etal. (2017), assuming that the wavefields of nearby points contribute to the approximates of the spatial derivatives of the central point and if the distances from nearby points to the center point are equal, the contributions are the same, we introduce a 9-point scheme to discretize the spatial derivatives. Take Eq.(1) for example:

$$\begin{aligned}\frac{\partial^2 u}{\partial x^2} \approx \frac{1}{\Delta x^2}&\left[c_0 u_{m,n} + c_1\left(u_{m-1,n}+u_{m+1,n}\right) + c_2\left(u_{m,n-1}+u_{m,n+1}\right)\right.\\&\left.+c_3\left(u_{m-1,n-1}+u_{m+1,n-1}+u_{m-1,n+1}+u_{m+1,n+1}\right)\right]\end{aligned}, \tag{5}$$

Table3
The Optimization coefficients for different Poisson's ratio σ with $r = \Delta x/\Delta z = 2$.

σ	0.1	0.2	0.3	0.4	0.45
b_0	0.5646	0.5426	0.5169	0.5132	0.5436
b_1	0.1196	0.1300	0.1483	0.1493	0.1383
b_2	0.1228	0.1335	0.1464	0.1489	0.1345
b_3	−0.0123	−0.0174	−0.0243	−0.0274	−0.0223
c_0	−1.8823	−1.7431	−4.5812	−1.3749	−1.2314
c_1	0.9535	0.8815	−1.7228	0.6899	0.6164
c_2	−0.0616	−0.1309	2.2923	−0.3134	−0.3845
c_3	0.0246	0.0605	0.8605	0.1555	0.1919
d_0	−1.2126	−1.1842	−0.3955	−1.0938	−1.0570
d_1	−0.3983	−0.4121	0.1994	−0.4549	−0.4723
d_2	0.6070	0.5927	−0.0527	0.5471	0.52861
d_3	0.1988	0.2057	0.0255	0.2274	0.2361