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A robust method of computing finite difference coefficients based on Vandermonde matrix

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A R T I C L E I N F O

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A B S T R A C T

When the finite difference (FD) method is employed to simulate the wave propagation, high-order FD method is preferred in order to achieve better accuracy. However, if the order of FD scheme is high enough, the coefficient matrix of the formula for calculating finite difference coefficients is close to be singular. In this case, when the FD coefficients are computed by matrix inverse operator of MATLAB, inaccuracy can be produced. In order to overcome this problem, we have suggested an algorithm based on Vandermonde matrix in this paper. After specified mathematical transformation, the coefficient matrix is transformed into a Vandermonde matrix. Then the FD coefficients of high-order FD method can be computed by the algorithm of Vandermonde matrix, which prevents the inverse of the singular matrix. The dispersion analysis and numerical results of a homogeneous elastic model and a geophysical model of oil and gas reservoir demonstrate that the algorithm based on Vandermonde matrix has better accuracy compared with matrix inverse operator of MATLAB.

1. Introduction

The finite difference method has been widely applied in the numerical simulation of seismic wave propagation (Tan and Huang, 2014), as its relatively straightforward implementation and flexible ability to deal with complex objectives. Alterman and Karal (1968) applied finite difference method into seismic numerical simulation in 1968. Alford et al. (1974) studied the accuracy of finite difference method and discussed the grid dispersion. Virieux (1984, 1986) investigated the SH and P-SV wave propagation by finite difference method based on velocity-stress wave equations.

Staggered-grid finite difference (SGFD) method (Pitarka, 1999) is an effective tool to increase the accuracy of numerical simulation, which means that the model parameters and variables are arranged at different grids. Staggered-grid finite difference method was first used by Graves (1996) in 1996 to simulate 3D elastic media. Moczo et al. (2000) discussed the stability and grid dispersion of fourth-order staggered-grid finite difference method. As its effectiveness, staggered-grid finite difference method has been applied into the wave propagation in complex media (Gao and Zhang, 2013; Zhang and Gao, 2014a; Zhang and Gao, 2014b).

In order to increase the accuracy of SGFD method, we can use higher-order SGFD method, or decrease the time step and the grid spacing. In addition, optimization methods are employed to calculate finite difference coefficients to achieve better accuracy (Wang et al., 2017; Yang et al., 2017), such as simulated annealing method (Zhang and Yao, 2013), least-squares method (Ren and Liu, 2014). Traditionally, high-order finite difference method is one of the most convenient ways to increase the accuracy, as it does not need extra operations at the interfaces.

We find that the FD method is not accurate when the FD order is high enough through the dispersion analysis if the FD coefficients are calculated by matrix inverse operator of MATLAB. The reason is that the coefficient matrix of the formula of calculating finite difference coefficients is close to be singular, but matrix inverse is used in the calculation. We realize that the coefficient matrix can be translated into a Vandermonde matrix. Then matrix inverse in the calculation of finite difference coefficients can be avoided, and is replaced by a recursive algorithm based on Vandermonde matrix (El-Mikkawy, 2003; Hassan et al., 2012). This algorithm has better accuracy than the way of matrix inverse operator in MATLAB.

This paper is organized as follows. The dispersion parameters and stability condition of elastic media are derived in Section 2. In Section 3, we give the algorithm based on Vandermonde matrix

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of computing finite difference coefficients. In Section 4, the numerical results of two methods of calculating finite difference coefficients are compared. Finally, we state the conclusion of this paper in Section 5.

2. Formulation

2.1. Wave equations in elastic media

We concentrate on constructing a finite difference scheme for the following elastic wave equations that have been widely used in seismic numerical simulation,

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{A}\frac{\partial \mathbf{u}}{\partial x} + \mathbf{B}\frac{\partial \mathbf{u}}{\partial y} \tag{1}$$

where the unknown vector is

$$\mathbf{u} = [v_x \ v_z \ \tau_{xx} \ \tau_{zz} \ \tau_{xz}]^T \tag{2}$$

The coefficient matrices are defined as

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & \frac{1}{\rho} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\rho} \\ \lambda+2\mu & 0 & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{\rho} \\ 0 & 0 & 0 & \frac{1}{\rho} & 0 \\ 0 & \lambda & 0 & 0 & 0 \\ 0 & \lambda+2\mu & 0 & 0 & 0 \\ \mu & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

where, (v_x, v_z) is the velocity components, $(\tau_{xx}, \tau_{xz}, \tau_{zz})$ is the stress components, λ and μ represent the Lame parameters, and ρ is the density.

2.2. Finite difference scheme

In staggered-grid finite difference scheme, second-order accuracy derivative in time domain is

$$\frac{\partial f}{\partial t} = \frac{1}{\tau}\left(f\left(t+\frac{\tau}{2}\right) - f\left(t-\frac{\tau}{2}\right)\right) + O(\tau^2) \tag{5}$$

And the $2M$-th order accuracy derivative in space domain is expressed as (Dong et al., 2000)

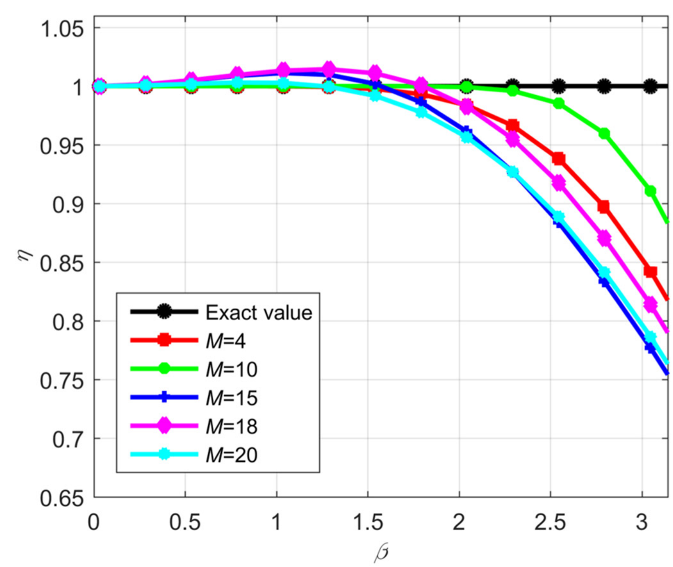
$$\frac{\partial f}{\partial x} = \frac{1}{h}\sum_{m=1}^{M} a_m \left\{ f\left[x+\frac{h}{2}(2m-1)\right] - f\left[x-\frac{h}{2}(2m-1)\right]\right\} + O\left(h^{2M}\right) \tag{6}$$

where, τ is the time step, h is the grid space, a_m $(m = 1,2,\ldots,M)$ are the staggered-grid finite difference coefficients.

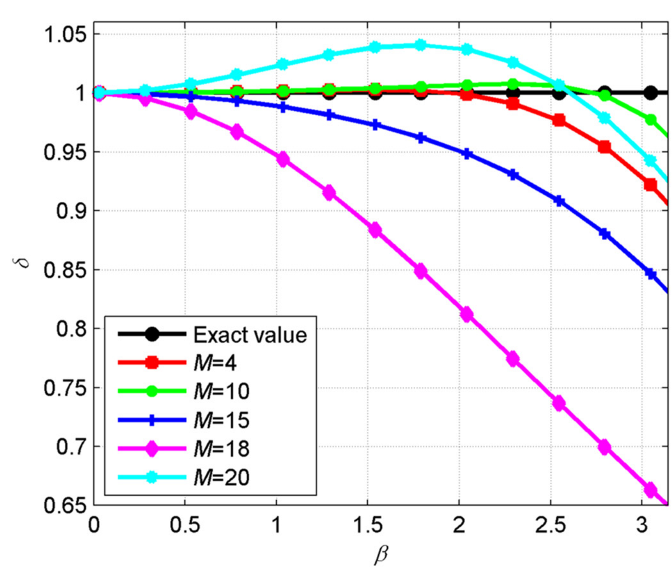
2.3. Numerical dispersion parameters

The numerical dispersion can be measured by the dispersion parameters. Based on Eq. (A11), the numerical dispersion can be represented by the numerical dispersion parameter which is related to spatial parameters

$$\eta_i = \frac{2\sum_{m=1}^{M} a_m \sin\left(\left(m-\frac{1}{2}\right)k_{x\_i}h\right)}{\hat{k}_{x\_i}h} \tag{7}$$



(a)



(b)

Fig. 1. Dispersion parameters (a) η and (b) δ varied with β for different FD orders. Here, $\tau = 0.001$ s, $h = 10$ m, the FD coefficients are calculated based on the matrix inverse operator of MATLAB.