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Inverts permittivity and conductivity with structural constraint in GPR FWI based on truncated Newton method

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ABSTRACT

Full waveform inversion (FWI) of ground penetrating radar (GPR) is a promising technique to quantitatively evaluate the permittivity and conductivity of near subsurface. However, these two parameters are simultaneously inverted in the GPR FWI, increasing the difficulty to obtain accurate inversion results for both parameters. In this study, I present a structural constrained GPR FWI procedure to jointly invert the two parameters, aiming to force a structural relationship between permittivity and conductivity in the process of model reconstruction. The structural constraint is enforced by a cross-gradient function. In this procedure, the permittivity and conductivity models are inverted alternately at each iteration and updated with hierarchical frequency components in the frequency domain. The joint inverse problem is solved by the truncated Newton method which considering the effect of Hessian operator and using the approximated solution of Newton equation to be the perturbation model in the updating process. The joint inversion procedure is tested by three synthetic examples. The results show that jointly inverting permittivity and conductivity in GPR FWI effectively increases the structural similarities between the two parameters, corrects the structures of parameter models, and significantly improves the accuracy of conductivity model, resulting in a better inversion result than the individual inversion.

1. Introduction

Ground penetrating radar (GPR) is a nondestructive geophysical technique utilized to investigate properties of the shallow subsurface. It has been used in many fields, such as civil engineering, environmental monitoring, archaeology and glaciology. Tomography technique for GPR data based on the inversion of first arrival times is an effective tool to delineate the geometry of subsurface or characterize the structure of buried objects. However, the resolution provided by the ray-based tomography is limited by using only the travel time kinematics, which causing insufficient resolution scale that approximately the diameter of first Fresnel zone (Williamson and Worthington, 1993; Ernst et al., 2007a). By contrast, full waveform inversion can take full use of waveforms, result in a higher resolution that of the order of half of wavelength. FWI was originally developed for the acoustic and elastic wave equations in the seismic exploration (Lailly, 1983; Tarantola, 1984; Pratt and Worthington, 1990; Pratt, 1990; Nuber et al., 2015), then is rapidly developed for Maxwell's equations to estimate the permittivity and conductivity of subsurface medium (Ernst et al., 2007b; Meles et al., 2010; Belina et al., 2012; Yang et al., 2013; Lavoue et al., 2014; Keskinen et al., 2017). In GPR FWI, permittivity and conductivity are simultaneously inverted, which increases the difficulty to construct reliable models for both parameters. Improving the accuracy of each parameter model is critical to improve the overall accuracy of inversions.

Joint inversion is an effective way to reduce the uncertainty of inversion and increase the accuracy of reconstructed models in the geophysical inverse problem. Two types of approaches are usually used to couple different geophysical parameters: one is to combine parameters through petrophysical relationship (Ghose and Slob, 2006; Gao et al., 2012), and the other is to couple parameters with structural constraint that enforced by a cross-gradient function (Gallardo and Meju, 2003, 2004; Meju et al., 2003). Joint inversion with structural constraint reduces the set of acceptable models by increasing the structural similarity between different parameter models. Gallardo and Meju (2003, 2004) firstly introduced the concept of cross-gradient function and jointly inverted the DC resistivity and seismic traveltime data. The cross-gradient function is widely used for the joint interpretation of different parameters or datasets, such as P-wave and S-wave velocities (Tryggvason and Linde, 2006), crosshole seismic and GPR data (Linde et al., 2008), conductivity and P-wave velocity (Hu et al., 2009; Shi et al., 2017), ERT and GPR traveltime data (Doetsch et al., 2010; Bouchedda et al., 2012). Meanwhile, the cross-gradient function also can be utilized to combine multiple geophysical datasets (Gallardo, 2007; Moorkamp et al., 2011; Abubakar et al., 2012; Zhu and Harris, 2015; Pak et al., 2017).

The local optimization method used to solve the inverse problem in GRP FWI is critical to obtain accurate results for both parameters. A

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mainly difference between these optimization methods is depending on the way to handle Hessian operator. The consideration of Hessian operator can improve the convergence rate of optimization process and correct the wave propagation effects related to double scattering (Pratt et al., 1998). The inverse Hessian operator is approximated in quasi-Newton method (Byrd et al., 1995), while the approximation may be inaccurate when multiple scattering cannot be neglected. Truncated Newton method can better account for the effects of Hessian operator. In the truncated Newton method, the approximate resolution of Newton equation is used to be a perturbation model in the updating process and solved by a conjugate gradient solver in the inner-loop of truncated Newton method (Nash, 2000; Nocedal and Wright, 2006). The computation of Hessian operator is replaced by the computation of Hessian-vector product which can be efficiently implemented through the second-order adjoint-state method (Wang et al., 1992; Fichtner and Trampert, 2011) or finite-difference method (Nash, 2000; Nocedal and Wright, 2006). Truncated Newton method has been used in the seismic FWI for the quantitative imaging of complex subsurface structures (Métivier et al., 2013, 2014). Pinard et al. (2015) investigated the potential of truncated Newton method in GPR FWI for estimation of permittivity and conductivity.

In this paper, I present a cross-gradient based GPR FWI to jointly invert permittivity and conductivity with structural constraint. The permittivity and conductivity are alternatively inverted at each iteration. The parameter models are constrained by the structure of each other through a weighted cross gradient term and updated with several frequencies in the frequency domain. The joint inversion procedure is based on the frame of truncated Newton method, which can consider the effects of Hessian operator in the optimization process. This joint inversion procedure will be tested by three synthetic examples. Then, the convergence behavior, values of cross-gradient term, and rms error of individual and joint inversion results will be discussed in the following section.

2. Methodology

2.1. Forward problem in the frequency domain

Restricting the Maxwell's equations to a 2D geometry results in two decoupled systems: the transverse electric mode (TE) and the transverse magnetic mode (TM). In the following, TE mode is utilized. Vibrating in the (XOZ) plane, for an electric dipole source oriented along the y-axis, leading to following scalar wave equations

$$\nabla^2 E(x,z,\omega) + \kappa^2 E(x,z,\omega) = -S(\omega), \tag{1}$$

where E is the electric field intensity (in V m^{-1}), $\kappa^2 = \omega^2\mu\varepsilon - i\omega\mu\sigma$, μ is the magnetic permeability (in H m^{-1}), ε is the dielectric permittivity (in F m^{-1}), σ is the electric conductivity (in S m^{-1}), $\omega = 2\pi f$ is the angular frequency (in rad s^{-1}), f is the frequency component (in Hz), and S is the electric source waveform in the frequency domain.

After spatial discretization with the 9-point frequency domain finite difference method (Jo et al., 1996), Eq. (1) can be written in a matrix form (Pratt and Worthington, 1990)

$$B(\varepsilon,\sigma,\omega)u = s(\omega), \tag{2}$$

where B is the complex-valued impedance matrix for the electromagnetic wavefield, u is the complex-valued electrical field, and s is the electric source term.

Eq. (2) can be solved by direct matrix factorization method such as lower-upper (LU) triangular decomposition (Golub and Loan, 1996). Perfectly matched layers (PML) are used to absorb the waves at the boundary of the computational domain (Bérenger, 1994).

2.2. Joint inverse problem

Define a joint objective function including both the individual objective function of GPR FWI and the cross-gradient term between permittivity and conductivity, such that

$$\Phi = \Phi_1 + \alpha\Phi_{cross}, \tag{3}$$

where Φ_1 is the individual objective function for GPR FWI, defined as

$$\Phi_1(\varepsilon,\sigma,\omega) = \frac{1}{2}\|d - u(\varepsilon,\sigma,\omega)\|^2, \tag{4}$$

where d is the observed GPR data, u is the simulated GPR data in the forward problem, $\alpha = \lambda\frac{\Phi_1(\varepsilon_0,\sigma_0)}{\Phi_{cross}(\varepsilon_0,\sigma_0)}$ is a normalizing factor, λ is a weighting factor ranging from [0, 1], ε_0 and σ_0 are initial models of permittivity and conductivity for current frequency component, Φ_{cross} is a cross-gradient term that coupling permittivity and conductivity with cross-gradient function (Gallardo and Meju, 2003, 2004), defined as

$$\Phi_{cross} = \frac{1}{2}(\nabla\varepsilon \times \nabla\sigma)^T(\nabla\varepsilon \times \nabla\sigma), \tag{5}$$

where T denotes transpose operator.

Then, alternately update permittivity and conductivity at each iteration (Ernst et al., 2007b). The updating process for these two parameters is similar. Therefore, I use a model, m, of dimension M to represents the permittivity model or the conductivity model that being updated in the following expressions.

The parameter model is updated through an iterative process, such that

$$m_{k+1} = m_k + \gamma_k \Delta m_k, \tag{6}$$

where Δm denotes the perturbation model, and γ denotes the scale length.

Within the framework of Newton algorithm, the perturbation model satisfy the equation defined as

$$\frac{\partial^2\Phi}{\partial {m_k}^2}\Delta m_k = -\frac{\partial\Phi}{\partial m_k}, \tag{7}$$

where $\frac{\partial^2\Phi}{\partial {m_k}^2} = H(m_k) + \alpha\frac{\partial^2\Phi_{cross}}{\partial {m_k}^2}$, and $\frac{\partial\Phi}{\partial m_k} = \nabla\Phi_1(m_k) + \alpha\frac{\partial\Phi_{cross}}{\partial m_k}$, $\nabla\Phi_1$ and H denote the gradient and Hessian matrix of the individual objective function for GPR FWI.

The computation of $\nabla\Phi_1$ can be performed efficiently through the first-order adjoint-state method (Plessix, 2006; Virieux and Operto, 2009; Lavoue et al., 2014), such that

$$\nabla\Phi_1(m_k) = \Re\left[u^T\left(\frac{\partial B}{\partial m_k}\right)^T B^{-1}\Delta d^*\right], \tag{8}$$

where $\Delta d = d - u$, the diffraction matrix $\frac{\partial B}{\partial m}$ (or sensitivity kernel) characterizes the sensitivity to the parameter m, that refers either to the permittivity or conductivity, $\Re$ denotes the real part operator, and $*$ denotes complex conjugate operator.

The computation of Hessian operator, H, is a time-consuming process that needs to compute M forward problems at least (Pratt, 1990), therefore it is sensible to avoid directly constructing the Hessian operator, which leading to the consideration of truncated Newton method.

2.3. Truncated Newton method

2.3.1. The flowchart of truncated Newton method

Truncated Newton method is a two-layer algorithm that divided into an outer-loop and an inner-loop. Parameter model is