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Refraction traveltime tomography based on damped wave equation for irregular topographic model

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A B S T R A C T

Land seismic data generally have time-static issues due to irregular topography and weathered layers at shallow depths. Unless the time static is handled appropriately, interpretation of the subsurface structures can be easily distorted. Therefore, static corrections are commonly applied to land seismic data. The near-surface velocity, which is required for static corrections, can be inferred from first-arrival traveltime tomography, which must consider the irregular topography, as the land seismic data are generally obtained in irregular topography.

This paper proposes a refraction traveltime tomography technique that is applicable to an irregular topographic model. This technique uses unstructured meshes to express an irregular topography, and traveltimes calculated from the frequency-domain damped wavefields using the finite element method. The diagonal elements of the approximate Hessian matrix were adopted for preconditioning, and the principle of reciprocity was introduced to efficiently calculate the Fréchet derivative. We also included regularization to resolve the ill-posed inverse problem, and used the nonlinear conjugate gradient method to solve the inverse problem.

As the damped wavefields were used, there were no issues associated with artificial reflections caused by unstructured meshes. In addition, the shadow zone problem could be circumvented because this method is based on the exact wave equation, which does not require a high-frequency assumption. Furthermore, the proposed method was both robust to an initial velocity model and efficient compared to full wavefield inversions. Through synthetic and field data examples, our method was shown to successfully reconstruct shallow velocity structures. To verify our method, static corrections were roughly applied to the field data using the estimated near-surface velocity. By comparing common shot gathers and stack sections with and without static corrections, we confirmed that the proposed tomography algorithm can be used to correct the statics of land seismic data.

1. Introduction

Land seismic data are strongly influenced by weathering layers and irregular topography, resulting in time-statics, which distort the recorded reflection signals over time and degrade event continuity; therefore, land seismic data require static correction, which helps in accurate interpretation of subsurface structure (Cox, 1999; Yilmaz, 2001). Generally, static correction requires information of near-surface velocity structures. In the case of simple media, traditional approaches, such as the slope/intercept method (Knox, 1967) and/or delay-time method (Barry, 1967), can be utilized. However, in the case of complex media, more sophisticated techniques are needed, such as refraction traveltime tomography (Chang et al., 2002).

Refraction traveltime tomography is an efficient and robust optimization approach that infers shallow subsurface velocities by fitting the modeled first-arrival traveltimes to the observed values. Full waveform inversion (FWI) is a similar technique for estimating subsurface velocities. However, unlike FWI, which uses both the amplitudes and traveltimes of the seismic data, refraction traveltime tomography employs the first-arrival traveltimes. Therefore, refraction traveltime tomography is less affected by noise in the seismic data and is more robust than FWI (Zelt and Chen, 2016). Refraction traveltime tomography is also a robust inversion algorithm in terms of the local minima problem even when using an initial velocity model that is far from the true model (Zhou et al., 1995). Hence, the inverted velocity model obtained from refraction traveltime tomography can be used as an initial velocity model for FWI, which suffers from cycle-skipping due to the absence of low frequency components (Alkhalifah and Choi, 2014).

Refraction traveltime tomography methods are generally categorized into ray-tracing, wavepath, Fresnel volume, and wave equation-based tomography (Zhang et al., 2014; Pyun et al., 2005). Ray-based tomography has been widely used because it is an efficient method. Consequently, a variety of ray-based tomography methods have been proposed (Hampson and Russell, 1984; Schneider and Kuo, 1985; Docherty, 1992; White, 1989; Zhu and McMechan, 1989; Stefani, 1995). However, ray-based tomography easily fails because ray-tracing assumes a high-frequency limit (Luo and Schuster, 1991). To

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solve the problem of high-frequency approximation, a variety of traveltime tomography methods that incorporate the band-limited nature of seismic data were introduced (Vasco et al., 1995). One such method is wavepath tomography, which calculates the Fréchet derivative based on the Born approximation (Vasco and Majer, 1993), and another is Fresnel volume tomography, which uses paraxial ray approximation (Červený and Soares, 1992). In addition to these two methods, wave equation-based tomography was introduced to resolve the problems associated with ray-tracing methods (Luo and Schuster, 1991). As the wave equation-based method does not suffer from the high-frequency assumption, it can handle high-velocity contrast models and avoid the shadow zone problem. These benefits have prompted the development of other variations of wave equation-based tomography. For example, Luo et al. (2016) suggested a full traveltime inversion method, which uses a similar algorithm to FWI, to estimate a kinematically accurate velocity model from traveltime information.

Here, we introduce a wave equation-based tomography method that can be applied to irregular topographic models. To deal with the irregular topography, the finite element method (FEM) is more appropriate to use than the finite difference method (FDM). Therefore, we adopted the wave equation-based tomography technique suggested by Pyun et al. (2005). This technique calculates the traveltimes and Fréchet derivatives from monochromatic damped wavefields using the FEM in the frequency domain. They adopted the principle of reciprocity between the sources and receivers to efficiently calculate the sensitivity (Shin et al., 2001).

Triangular meshes are generated using the mesh generator designed by Shewchuk (2002) to construct elements, and then the frequency-domain wave equation is solved using the FEM. From the modeled wavefields, the first-arrival traveltimes are calculated and used to perform refraction traveltime tomography. To check the accuracy of the proposed algorithms, the results of traveltime calculations were compared with those from a fast-marching eikonal solver and analytical solution. Our tomography method was also compared with the algorithm of Pyun et al. (2005). The proposed tomography method was verified through both synthetic and field data examples with irregular topography. Especially, for the field data example, static corrections were additionally performed to illustrate the reliability of our algorithm.

2. Traveltime calculation using the frequency-domain wave equation

2.1. Solutions to the wave equation using the FEM in the frequency domain

The wave equation must first be solved to calculate the wave equation-based traveltimes (Shin et al., 2002; Son et al., 2016). To obtain numerical solutions for the wave equation, the FDM and/or FEM are commonly used. However, the FDM is unsuitable for an irregular topographic model. Therefore, the FEM with unstructured mesh was chosen to represent an irregular topography. The wave equation in the frequency domain using the FEM is given as follows (Marfurt, 1984):

$$\mathbf{S}\tilde{\mathbf{u}} = \mathbf{f}, \tag{1}$$

with

$$\mathbf{S} = \mathbf{K} - \omega^2\mathbf{M}, \tag{2}$$

where $\mathbf{K}$ is the stiffness matrix, $\mathbf{M}$ is the mass matrix, $\tilde{\mathbf{u}}$ is the Fourier transformed wavefield vector, $\mathbf{f}$ is the source vector, and ω is the real angular frequency.

We used the triangular mesh generator designed by Shewchuk (2002), which is based on the Delaunay triangulation theory (Delaunay, 1934). To construct the irregular surface model, it is important to determine the surface elevation of the target area. The surface elevation can be extracted either from the trace header of the SEGY file or from additional measurements. Once the elevation is obtained, triangular elements are generated based on the user-defined reference grid spacing and the extracted elevation. The reference grid spacing is defined as the distance between adjacent nodal points at boundaries of the subsurface model. In this study, the unstructured mesh is generated with some constraints of minimum internal angle and maximum area of a triangle. We assigned a value of 30° for the minimum internal angle and the maximum area was set as half of the reference grid spacing squared.

With regard to the optimal grid spacing, the dispersion error is qualitatively considered. Care is required, as the dispersion error is related to the accuracy of calculated traveltimes (Shin et al., 2002), particularly when using the unstructured mesh. If a regular mesh with triangular elements is employed, dispersion error can be analyzed quantitatively. For examples, Liu et al. (2012) explained the dispersive behavior of the spectral element method using 'X' type triangular mesh. Mazzieri and Rapetti (2012) also described dispersion and dissipation errors within triangular elements divided from square elements. However, dispersion analysis for an unstructured mesh is generally impossible (Shao-Lin et al., 2014). In this study, therefore, the reference grid spacing is used to control the dispersion error. The reference grid spacing is determined by the relationship between grid spacing and optimum angular frequency, as reported by Shin et al. (2002).

2.2. Traveltime calculation using a damped wavefield

As mentioned in the Introduction, various methods of traveltime calculation based on the wave equation have been suggested. The method proposed by Shin et al. (2003) was adopted in this paper. Although the traveltime was actually calculated from the frequency-domain wavefields, this paper starts with time-domain wavefields for ease of understanding as follows:

$$u(x,y,z,t) = \tilde{A}(x,y,z)\delta(t - \tau(x,y,z)) \tag{3}$$

where t and τ are the time variable and the first-arrival traveltime, respectively, u indicates the time-domain wavefields, and δ represents the Dirac delta function. $\tilde{A}(x,y,z)$ is the amplitude of the damped wavefield with an attenuation coefficient ε, which can be written with the original amplitude, $A(x,y,z)$, and an exponential function as follows:

$$\tilde{A}(x,y,z) = A(x,y,z)e^{-\varepsilon t}, \tag{4}$$

where ε is calculated from the empirical relationship obtained by dispersion analysis (Shin et al., 2003). Through Eq. (4), the seismic signals and multiples can be attenuated at a late time point, and a first-arrival event is finally obtained (Fig. 1).

By taking the Fourier transform of Eq. (3), the Fourier transformed wavefields are obtained as follows:

$$\tilde{u}(x,y,z,\omega) = \tilde{A}(x,y,z)e^{-i\omega\tau(x,y,z,\omega)}, \tag{5}$$

where ω should be sufficiently small to avoid the wrap-around effect (Shin et al., 2003). Although ω should vary depending on the traveling distance and velocity, it is generally around 0.1–0.01. By taking the logarithm of Eq. (5), isolating the imaginary part, and dividing by ω, the first-arrival traveltimes can be obtained as follows:

$$\tau(x,y,z,\omega) = -\frac{1}{\omega}\,\mathrm{Im}[\,\ln\{\tilde{u}(x,y,z,\omega)\}]. \tag{6}$$

In practice, to compute the frequency-domain damped wavefields $u(x,y,z,\omega)$ in Eq. (6), a complex angular frequency ω^* is used as follows:

$$\omega^* = \omega + i\varepsilon. \tag{7}$$

As shown in Eq. (7), the complex angular frequency consists of a real angular frequency and a wrap-around suppression factor in the real and