ARTICLE IN PRESS

APPGEO-03369; No of Pages 11

Journal of Applied Geophysics xxx (2017) xxx–xxx

Contents lists available at ScienceDirect

Journal of Applied Geophysics

journal homepage: www.elsevier.com/locate/jappgeo

Acoustic propagation operators for pressure waves on an arbitrarily curved surface in a homogeneous medium

Yimin Sun [a,*], Eric Verschuur [b], Roald van Borselen [a]

[a] EXPEC ARC GRC Delft, Aramco Overseas Company BV, The Netherlands
[b] Delft University of Technology, The Netherlands

ARTICLE INFO

Article history:
Received 4 June 2017
Received in revised form 13 September 2017
Accepted 9 November 2017
Available online xxxx

ABSTRACT

The Rayleigh integral solution of the acoustic Helmholtz equation in a homogeneous medium can only be applied when the integral surface is a planar surface, while in reality almost all surfaces where pressure waves are measured exhibit some curvature. In this paper we derive a theoretically rigorous way of building propagation operators for pressure waves on an arbitrarily curved surface. Our theory is still based upon the Rayleigh integral, but it resorts to matrix inversion to overcome the limitations faced by the Rayleigh integral. Three examples are used to demonstrate the correctness of our theory – propagation of pressure waves acquired on an arbitrarily curved surface to a planar surface, on an arbitrarily curved surface to another arbitrarily curved surface, and on a spherical cap to a planar surface, and results agree well with the analytical solutions. The generalization of our method for particle velocities and the calculation cost of our method are also discussed.

1. Introduction

The homogeneous acoustic wave equation,

$$\nabla^2 p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}, \tag{1}$$

where p represents the pressure waves and c represents the wave propagation velocity in the medium, is the foundation of almost all wavefield extrapolation in geophysics. Although the homogeneous acoustic wave equation appears simple, its solution is not trivial at all. Numerically several methods exist to solve Eq. (1), for instance finite-difference time-domain method (Alford et al., 1974) and finite element method (Chopra et al., 1969), and the situation of complex topography can also be addressed with some modifications to these numerical methods, such as finite-difference based on vacuum formulation (Pitarka and Irikura, 1996), discontinuous-grid finite-difference method (Hayashi et al., 2001), staggered discontinuous Galerkin method (Chung et al., 2015) and grid-characteristic method (Shragge, 2014). Analytically, in a homogeneous medium the solution of Eq. (1) is either the Kirchhoff integral or the Rayleigh integral. Details of derivation of Kirchhoff integral and Rayleigh integral can be found in many classical text books, e.g. Berkhout (1985, 1987) and Gisolf and Verschuur (2010), and in Appendix A some key information about these integrals are also listed. The Kirchhoff integral requires pressure waves, particle velocities, Green's functions and the spatial derivatives of the Green's functions to be known along a closed surface encircling the target point where the wavefield reconstruction is to take place. However, in reality it is not practical to have all these conditions satisfied simultaneously. In contrast to the Kirchhoff integral, the Rayleigh II integral only requires pressure waves and derivatives of Green's functions in order to reconstruct the pressure wavefield at the target point, but this information has to be available on a planar surface. In real data acquisition, due to both economic and practical reasons, normally pressure waves can only be acquired on a realistically curved surface, hence neither the Kirchhoff integral nor the Rayleigh II integral can be correctly computed. As a tradeoff solution, it is common practice to ignore the surface curvature when using the Rayleigh II integral (see e.g. Berkhout, 1985), which is an approximation referred to as "the traditional solution" in this paper. This compromise results in unwanted inaccuracies in wavefield extrapolation as the Rayleigh theory is actually violated, so it is still an open question how pressure wavefield extrapolation operators can be accurately built on a curved surface.

2. Theory

The Rayleigh II integral (see e.g. Gisolf and Verschuur, 2010) reads as follows:

$$P(\boldsymbol{r}_A) = -2\int_{S_0\to\infty} P\nabla G\cdot \boldsymbol{n} dS_0 \tag{2}$$

where P represents pressure waves of a certain frequency, S_0 is an infinite planar integral surface, $\boldsymbol{r}_A$ is the target location where the pressure wavefield needs to be reconstructed, G is the Green's function from a

* Corresponding author.
E-mail address: yimin.sun@aramcooverseas.com (Y. Sun).

https://doi.org/10.1016/j.jappgeo.2017.11.007
0926-9851/

Please cite this article as: Sun, Y., et al., Acoustic propagation operators for pressure waves on an arbitrarily curved surface in a homogeneous medium, Journal of Applied Geophysics (2017), https://doi.org/10.1016/j.jappgeo.2017.11.007

certain point on the infinite planar integral surface to the target location $\boldsymbol{r}_A$, and $\boldsymbol{n}$ is the unit normal vector at G's location on S_0. Eq. (2) actually carries three important messages regarding the pressure wavefield extrapolation. Firstly, although the integral surface has to be a planar surface, there is no limitation for the target surface. Secondly, the Rayleigh II integral is capable of handling both forward and backward wavefield extrapolation as long as the corresponding Green's functions are provided. Thirdly, the pressure wavefield extrapolation process is a linear system.

For the most general P-wave extrapolation situation, i.e. to propagate pressure waves acquired on an arbitrarily curved surface to another curved surface, as shown in Fig. 1(a), we propose to carry it out in two steps, as shown in Fig. 1(d). By introducing an intermediate planar surface, the complete wave propagation process can now be divided into pressure waves propagating from a curved surface to the intermediate planar surface and then the pressure waves propagating from the intermediate planar surface to the target curved surface. Since the second step is exactly what is described by the Rayleigh II integral, only the first step needs our attention. Knowing that wavefield extrapolation via the Rayleigh II integral is a linear system, we assume that the propagation of pressure waves from a curved surface to a planar surface is also a linear system, and the corresponding propagation operator matrix is denoted as $\boldsymbol{B}$. As illustrated in Fig. 1(b) we assume that in the frequency domain there exists a $\boldsymbol{B}$ such that pressure waves on the planar surface, w, are related to pressure waves on the curved surface, u, via the equation $w = \boldsymbol{B}u$. Note that w and u are vectors, and how 3D wavefields are packed into vectors will be explained later on. However, neither the Kirchhoff theory nor the Rayleigh theory tells how to straightforwardly build $\boldsymbol{B}$, and hence it has to be calculated via a detour. The Rayleigh II integral actually tells that u and w can be related via another relationship $u = \boldsymbol{A}w$ as w is pressure waves measured on a planar surface while $\boldsymbol{A}$ is the corresponding operator matrix for pressure waves as shown in Fig. 1(c). If these two relationships are compared, i.e. $w = \boldsymbol{B}u$ and $u = \boldsymbol{A}w$, and if we further assume that data sampling on both surfaces is the same, i.e. $\dim(u) = \dim(w)$, it can then be easily realized that the operator matrix $\boldsymbol{B}$ indeed exists in theory, and is given by

$$\boldsymbol{B} = \boldsymbol{A}^{-1}. \tag{3}$$

According to Eq. (3), to build the operator matrix $\boldsymbol{B}$, we need to first use the Rayleigh II integral to build its corresponding operator matrix $\boldsymbol{A}$, and then the inverse of $\boldsymbol{A}$ gives the $\boldsymbol{B}$ that is needed. This implies that the forward propagation matrix $\boldsymbol{B}$ needs the backward Rayleigh II integral to build $\boldsymbol{A}$, while the backward propagation matrix $\boldsymbol{B}$ needs the forward Rayleigh II integral to build $\boldsymbol{A}$. Moreover, evanescent waves should be handled properly in order for correct physics to be reflected in matrix $\boldsymbol{B}$. In this paper, we demonstrate step by step how to build a 3D forward propagation matrix $\boldsymbol{B}$ of pressure waves from an arbitrarily curved surface to a planar surface, and the backward propagation matrix $\boldsymbol{B}$ can be built by following exactly the same route.

We resort to 3D FK operator theory (see e.g. Berkhout, 1985, 1987 and Blacquiere, 1989) as the mathematical framework for us to build $\boldsymbol{A}$, and in the Appendix B more detailed information about this well-established theory can be found. In the FK (wavenumber-frequency domain) theory, starting from a planar surface $z = z_l$, the solution of the acoustic Helmholtz equation can be written as

$$\tilde{P}(k_x, k_y, z, \omega) = \exp(-jk_z|z-z_l|)\tilde{P}(k_x, k_y, z_l, \omega), \tag{4}$$

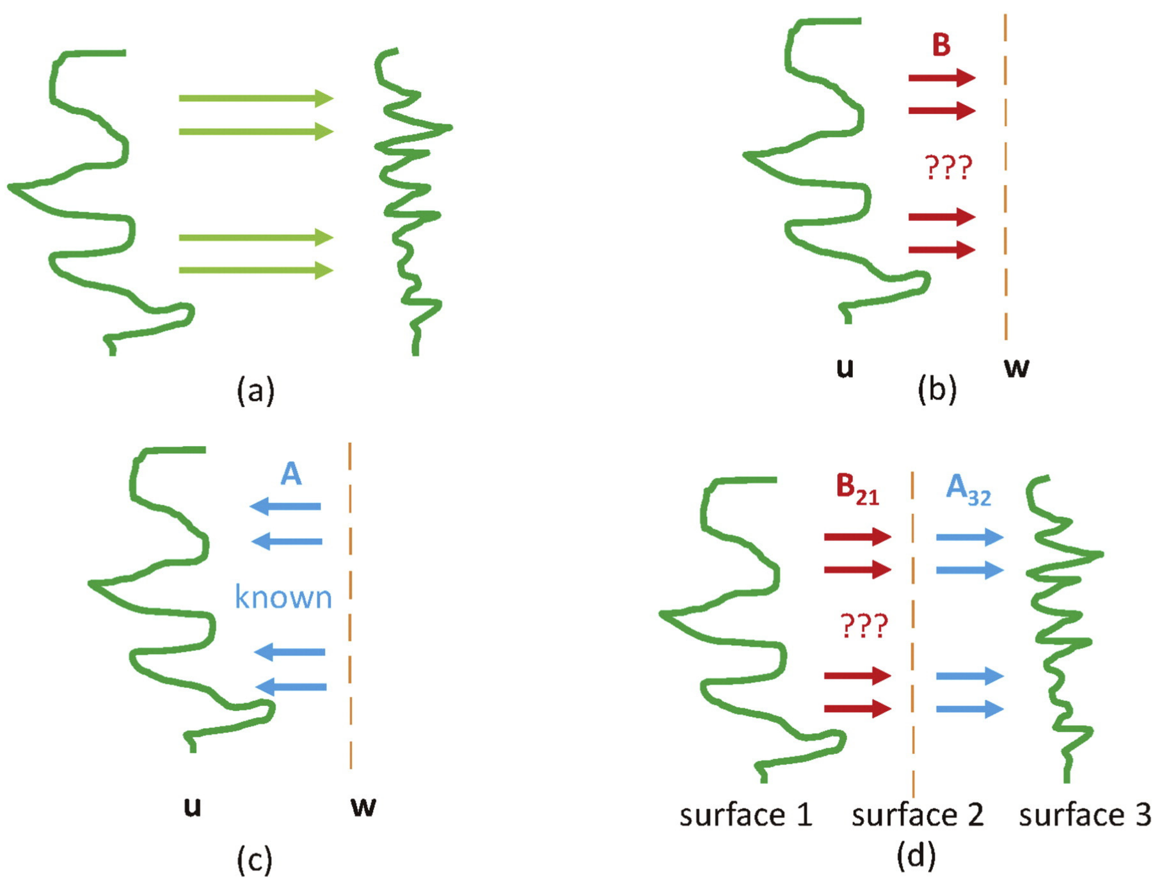

Fig. 1. (a) Propagation of pressure waves from a curved surface to another curved surface. (b) Operator matrix $\boldsymbol{B}$ satisfying $w = \boldsymbol{B}u$. (c) Operator matrix $\boldsymbol{A}$ satisfying $u = \boldsymbol{A}w$. Note here we already assume that the data sampling is the same on both surfaces. (d) Propagation after introducing an intermediate planar surface to divide the complete propagation process into two phases, and the corresponding propagation matrices are $\boldsymbol{B}_{21}$ and $\boldsymbol{A}_{32}$.