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A developed nearly analytic discrete method for forward modeling in the frequency domain

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ABSTRACT

High-efficiency forward modeling methods play a fundamental role in full waveform inversion (FWI). In this paper, the developed nearly analytic discrete (DNAD) method is proposed to accelerate frequency-domain forward modeling processes. We first derive the discretization of frequency-domain wave equations via numerical schemes based on the nearly analytic discrete (NAD) method to obtain a linear system. The coefficients of numerical stencils are optimized to make the linear system easier to solve and to minimize computing time. Wavefield simulation and numerical dispersion analysis are performed to compare the numerical behavior of DNAD method with that of the conventional NAD method. The results demonstrate the superiority of our proposed method. Finally, the DNAD method is implemented in frequency-domain FWI, and high-resolution inverse results are obtained.

1. Introduction

Full waveform inversion (FWI) is a hot topic in geophysical research. Computational mathematicians and geophysicists produced outstanding work in past decades (Lailly, 1983; Tarantola, 1984; Kolb et al., 1986; Mora, 1987), establishing foundation for the development of high-resolution seismic imaging technology. FWI was first studied in the time domain (Tarantola, 1986). In the 1990s, Pratt et al. (1998) proposed performing FWI in the frequency domain and established the corresponding inversion theory (Pratt, 1999). The frequency-domain FWI has many advantages, which are discussed in detail by Song and Williamson, 1995 and Lang and Yang (2017).

Forward modeling is known to be a critical aspect of FWI. The major operations and computing time costs of waveform inversion are associated with the forward modeling processes. Thus, the efficiency of FWI is largely influenced by the forward modeling algorithms. When performing forward modeling in the frequency domain, one should first select proper numerical schemes to discretize the frequency-domain wave equations and implement absorbing boundary conditions, such as perfect matched layers (PML) (Komatitsch and Tromp, 2003), at the boundary of the computing area. Then, a large sparse linear algebraic system is obtained. An efficient method for solving the linear system is necessary. Thus, a complete set of frequency-domain forward algorithms includes a spatial discretization scheme, an absorbing boundary condition and an effective solver for the linear system.

The commonly used numerical schemes for numerical modeling in the geophysical community include finite difference methods (Kelly et al., 1976; Igel et al., 1995; Liu et al., 2014a), finite element methods (Lysmer and Drake, 1972; Marfurt, 1984; Liu et al., 2014b), pseudo-spectral methods (Kosloff and Baysal, 1982), and spectral element methods (Seriani and Priolo, 1994; Komatitsch et al., 2005). Finite difference methods have been widely used due to their easy implementation, high computational efficiency, low memory costs and high parallelism. The nearly analytic discrete (NAD) method is a novel finite difference method. It possesses most of the merits of conventional finite difference methods but has additional advantages, including the ability to more easily suppress numerical dispersion, shorter operator length and the ability to provide more wavefield information (Yang et al., 2003, 2004, 2006). Furthermore, dispersion analysis shows that the numerical velocity associated with the NAD method is closer to the physical velocity of seismic wave propagation compared with other classical numerical schemes (Yang et al., 2010, 2012). The NAD method was first introduced in frequency-domain forward modeling by Lang and Yang (2017). The frequency-domain NAD method also has the ability to suppress numerical dispersion and save computing time, resulting in properties similar to those properties in the time domain.

When using a numerical scheme to discretize the frequency-domain wave equations, the linear system should be solved efficiently. Nevertheless, this process is not easy due to the special properties of the coefficient matrix (also called the impedance matrix) (Pratt, 1999). In most cases, the impedance matrix is large in scale and has a certain sparse structure. The values of its elements are complex due to absorbing boundary conditions and are dependent on the spatial interval size

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and frequency. If the frequency values are large enough, the impedance matrix become indefinite (its eigenvalues are located at both sides of imaginary on the complex plane), ill conditioned and even close to a singular matrix. Such factors make it difficult to solve the linear system, and many conventional methods usually work poorly. The methods for solving this linear system can be divided into two categories: direct methods and iteration methods. The direct methods are based on LU factorization with node reordering techniques (George, 1977). The iteration methods include preconditioned Krylov methods (Saad, 2003; Lang and Ren, 2015), multigrid methods (Kim and Kim, 2002; Plessix, 2007) and domain decomposition methods (Hagstrom et al., 1988; Gander et al., 2007). Each approach has its advantages and intrinsic drawbacks, which have been analyzed in detail by Lang and Yang (2017). Notably, a class of inexact rotated block triangular (IRBT) preconditioners (Lang and Ren, 2015) associated with Krylov subspace methods, such as GMRES (Saad and Schultz, 1986) and BiCGSTAB (Sleijpen and Fokkema, 1993), can solve this type of linear system effectively. Some numerical results show the superiority of such preconditioners with respect to other convention methods (Lang and Yang, 2017).

The acceleration of the frequency-domain forwarding modeling processes can be usually achieved according to three approaches: (i) Using high-precision numerical schemes to discretize the wave equation. High-precision numerical schemes can more easily suppress numerical schemes and the discrete grids can be coarser (Yang et al., 2003, 2006). Thus, the problem scale decreases, and computing time can be saved. (ii) Constructing more effective methods to solve the linear system. Solving the linear system effectively is a well-known challenge for FWI in the frequency domain. Therefore, an efficient method for solving the linear system is critical. (iii) Making the linear system easier to solve. To maintain a certain precision, we decrease the condition number of the impedance matrix by developing numerical schemes, and the corresponding linear system becomes easier to solve. Thus, the solving process costs less computing time.

In this work, the approach (iii) described above is adopted to accelerate the frequency-domain forward modeling. Based on the NAD method, we adjust the coefficients of the numerical stencils to obtain the DNAD method. When using the DNAD method to discretize the wave equations, the main diagonal part of the impedance matrix is more concentrated, and the corresponding condition number is lower. Thus, the process of solving the linear system is more stable and costs less computing time. To some extent, this concept is similar to constructing preconditioned iteration methods to solve the linear system. Both of these ideas involve decreasing the condition number of the impedance matrix and reducing computational costs in the solving processes.

Here, we first derive the DNAD method based on the ordinary fourth-order NAD method. Then, the DNAD method is used to discretize frequency-domain wave equations with the PML absorbing boundary condition. We analyze the properties of its coefficient matrix in detail and propose to use IRBT preconditioners associated with the GMRES method to solve the linear system. Wavefield simulation and numerical dispersion analysis are implemented to examine the numerical behavior of the DNAD method. We also perform FWI in the frequency domain based on the DNAD method for a two-layer medium model. The numerical results illustrate the effectiveness of our proposed method.

2. Frequency-domain wave equation

Consider the following two-dimensional frequency-domain acoustic wave equation in a constant-density medium:

$$\Delta u(x,z)+\frac{\omega^2}{c^2}u(x,z)=-\frac{1}{c^2}s,\quad (x,z)\in D, \tag{1}$$

where Δ is the two-dimensional Laplace operator, $\omega=2\pi f$ denotes the angular frequency, $c(x,z)$ is the velocity of acoustic wave propagation, s is the seismic source term and D denotes the two-dimensional computing area. We introduce the general approach for the numerical schemes based on the NAD method to discretize Eq. (1). Firstly, the partial derivative forms of Eq. (1) need to be derived with respect to x and z, which yield the following:

$$\begin{cases}\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial z^2}+\dfrac{\omega^2}{c^2}u=-\dfrac{1}{c^2}s\\[2mm] \dfrac{\partial^2 u_x}{\partial x^2}+\dfrac{\partial^2 u_x}{\partial z^2}+\dfrac{\omega^2}{c^2}u_x=-\dfrac{1}{c^2}\dfrac{\partial s}{\partial x}\\[2mm] \dfrac{\partial^2 u_z}{\partial x^2}+\dfrac{\partial^2 u_z}{\partial z^2}+\dfrac{\omega^2}{c^2}u_z=-\dfrac{1}{c^2}\dfrac{\partial s}{\partial z}\end{cases} \tag{2}$$

Note that Eq. (2) contains the actual discrete partial differential equations for all types of NAD methods.

To eliminate the influence of reflected waves from artificial boundaries, we should consider an absorbing boundary condition. Here, we adopt the PML boundary condition (Komatitsch and Tromp, 2003), which is one of the most widely used absorbing boundary conditions. The x-direction is taken as example to derive the absorbing boundary condition. The relationship between the complex coordinate $\tilde{x}$ in the PML region and the real coordinate x is as follows:

$$\tilde{x}=x-\frac{i}{\omega}\int_0^x d_x(p)dp, \tag{3}$$

where $d_x(x)>0$ is the attenuation function, and $i=\sqrt{-1}$ denotes the imaginary unit. From Eq. (3), we obtain the following equation.

$$\frac{\partial x}{\partial \tilde{x}}=\frac{1}{1-\dfrac{i}{\omega}d_x(x)}. \tag{4}$$

The choice of $d_x(x)$ is from Komatitsch and Tromp (2003):

$$d_x(x)=-\frac{3a}{2\delta}\log(R)\left(\frac{x}{\delta}\right)^2, \tag{5}$$

where δ is the width of the PML layer, a is the acoustic wave speed in the PML region, and R is the theoretical reflection coefficient after discretization, which can be chosen to be a small constant (typically 10^{-3}).

After replacing x and z with $\tilde{x}$ and $\tilde{z}$, the partial differential equations in Eq. (2) can be rewritten as follows:

$$\begin{cases}\dfrac{\partial^2 u}{\partial \tilde{x}^2}+\dfrac{\partial^2 u}{\partial \tilde{z}^2}+\dfrac{\omega^2}{c^2}u=-\dfrac{1}{c^2}s\\[2mm] \dfrac{\partial^2 u_{\tilde{x}}}{\partial \tilde{x}^2}+\dfrac{\partial^2 u_{\tilde{x}}}{\partial \tilde{z}^2}+\dfrac{\omega^2}{c^2}u_{\tilde{x}}=-\dfrac{1}{c^2}\dfrac{\partial s}{\partial x}\\[2mm] \dfrac{\partial^2 u_{\tilde{z}}}{\partial \tilde{x}^2}+\dfrac{\partial^2 u_{\tilde{z}}}{\partial \tilde{z}^2}+\dfrac{\omega^2}{c^2}u_{\tilde{z}}=-\dfrac{1}{c^2}\dfrac{\partial s}{\partial z}\end{cases} \tag{6}$$

Next, $\tilde{x}$ and $\tilde{z}$ in Eq. (6) need to be changed into x and z according to Eq. (4). The detailed derivation based on (Lang and Yang, 2017). To avoid repetition, we present the frequency-domain wave equations