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Identification of different geologic units using fuzzy constrained resistivity tomography

Anand Singh *, S.P. Sharma

Department of Geology and Geophysics, Indian Institute of Technology Kharagpur, 721302, West Bengal, India

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ABSTRACT

Different geophysical inversion strategies are utilized as a component of an interpretation process that tries to separate geologic units based on the resistivity distribution. In the present study, we present the results of separating different geologic units using fuzzy constrained resistivity tomography. This was accomplished using fuzzy c means, a clustering procedure to improve the 2D resistivity image and geologic separation within the iterative minimization through inversion. First, we developed a Matlab-based inversion technique to obtain a reliable resistivity image using different geophysical data sets (electrical resistivity and electromagnetic data). Following this, the recovered resistivity model was converted into a fuzzy constrained resistivity model by assigning the highest probability value of each model cell to the cluster utilizing fuzzy c means clustering procedure during the iterative process. The efficacy of the algorithm is demonstrated using three synthetic plane wave electromagnetic data sets and one electrical resistivity field dataset. The presented approach shows improvement on the conventional inversion approach to differentiate between different geologic units if the correct number of geologic units will be identified. Further, fuzzy constrained resistivity tomography was performed to examine the augmentation of uranium mineralization in the Beldih open cast mine as a case study. We also compared geologic units identified by fuzzy constrained resistivity tomography with geologic units interpreted from the borehole information.

1. Introduction

Electrical Resistivity Tomography (ERT) and plane-wave diffusive electromagnetic techniques (i.e., direct current, very low frequency, and magnetotelluric method) are among the popular methods in geophysics to map both shallow (direct current, very low frequency, radio magnetotelluric method) and deep resistivity features (magnetotellurics) on a large scale. Inversion provides a mathematical framework to obtain a reliable resistivity model of the earth's subsurface from these data sets. Though there are a finite number of data sets, the resistivity distribution of the earth is continuous. Thus, there are many subsurface resistivity models that can fit the observed data equally well due to the non-uniqueness of the geophysical inverse problem. Over the last 30 years, many resistivity inversion algorithms have been developed to image complex geological structures with different scales (e.g., Constable et al., 1987; Siripunvaraporn and Egbert, 2000; Rodi and Mackie, 2001; Lee et al., 2009; Kamm and Pedersen, 2014; Kelbert et al., 2014; Zhou et al., 2014; Cockett et al., 2015; Singh and Sharma, 2015; Nittinger and Becken, 2016; Singh and Sharma, 2016).The aforementioned inversion algorithms have their own originality in terms of computation time and provide a smooth resistivity model which fit the observed data to the desired degree. However, in general, the boundaries between different geologic units are not clear because of less contrast in the recovered resistivity model. This makes the geologic interpretation based on subsurface resistivity distribution difficult. To overcome this, Mehanee and Zhdanov (2002) created blocky resistivity structure using magnetotelluric data.

To reduce the non-uniqueness in the resistivity distribution of subsurface, many researchers began dealing with joint inversion. Vozoff and Jupp (1975) demonstrated the first result based on joint inversion of magnetotelluric and direct current resistivity data. Joint inversion algorithms with other geophysical methods were introduced to further enhance the quality of the resistivity images (for e.g., Sasaki, 1989; Dobroka et al., 1991; Verma and Sharma, 1993; Meju, 1996; Candansayar and Tezkan, 2008; Sharma and Verma, 2011). Moorkamp et al. (2011) developed 3D joint inversion of gravity, magnetotelluric, and seismic data over a marine salt dome. Constraining geophysical inversion with additional independent information and joint inversion of various geophysical data sets are an active area of research in the geophysical community.

In general, we first acquired the resistivity model through inversion and based on that resistivity model, classified distinct geologic units as a post-inversion process. For example, Bedrosian et al. (2007) independently obtained the resistivity model from magnetotelluric data and

* Corresponding author.
E-mail addresses: anand@gg.iitkgp.ernet.in (A. Singh), spsharma@gg.iitkgp.ernet.in (S.P. Sharma).

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the velocity model from seismic data. Further, they identified the statistical correlation between resistivity and seismic velocity in a joint parameter space and drew a lithology-derived empirical relationship between the resistivity and velocity models. Infante et al. (2010) combined geospectral images and geophysical signatures after the joint inversion of seismic and direct current resistivity data to obtain a better boundary between two geologic units. Kalscheuer et al. (2013) obtained a 2D resistivity model from joint inversion of radio-magnetotelluric method / controlled source audio-magnetotelluric method and direct current gradient resistivity data. Further, based on the resistivity distribution, they interpreted structural boundaries of marine clay, dry soil, quick clay, alum shale and limestone under geotechnical constraints. Di Giuseppe et al. (2014) used the k-means clustering technique as the post-inversion process after obtaining the resistivity and P-wave velocity models from the independent individual inversion. Further, they identified five clusters in the cross-plot of resistivity and velocity section. After obtaining two distinct local relationships between electrical resistivity and seismic velocities, the hanging and footwall zones of the fault were identified. Ward et al. (2014) used the fuzzy c means clustering technique as a post-inversion process, which significantly improved the resistivity image.

In most geologic scenarios, different geologic units often have a distinct range of resistivity values (Reynolds, 1997). As a result, these geologic units can be classified by incorporating inverted resistivity values so that statistically similar resistivity values are grouped into one unit or cluster. In this manner, geologic separation can be accomplished with the help of the clustering procedure of resistivity distribution. Similar to Ward et al. (2014), the present study used the fuzzy c means clustering technique to differentiate between two geologic units. An attempt was made to incorporate the fuzzy c means clustering technique directly into the inversion framework to distinguish between two geologic units. The algorithm has been utilized on three synthetic data sets, viz., Very Low Frequency- Electromagnetic (VLF-EM), Very Low Frequency-Resistivity (VLF-R), and magnetotelluric (MT) method. The field Direct Current Resistivity (DCR) data showed the efficacy of the present study to differentiate the subsurface geologic units.

2. Theoretical background

2.1. Inversion approach

To obtain the reliable resistivity model from the DCR/MT/VLF data sets, we formulated our inverse problem using the Tikhonov objective function (Tikhonov and Arsenin, 1977):

$$\min_{m} \phi(\boldsymbol{m}) = \left\| \boldsymbol{W_d}\left(\boldsymbol{d^{obs}} - \boldsymbol{d^{pre}}\right)\right\|^2 + \lambda \|\boldsymbol{W_m m}\|^2, \tag{1}$$

$$\text{s.t. } m_i^{\min} < m_i < m_i^{\max} \text{ for some or all } i, \tag{2}$$

where, $m_i^{\min}$ and $m_i^{\max}$ are the lower and upper geological constrains for the ith resistivity model parameter, $\|\boldsymbol{W_d}(\boldsymbol{d^{obs}} - \boldsymbol{d^{pre}})\|^2$ is a data misfit term, and $\|\boldsymbol{W_m m}\|^2$ is a model regularization term, and λ is a Lagrange multiplier or regularization parameter and it will try to balance between the data misfit term and the model regularization term. $\boldsymbol{d^{obs}}$ is observed data sets (DCR/MT/VLF) and $\boldsymbol{d}^{pre}$ is a predicted or computed data. $\boldsymbol{W_d}$ and $\boldsymbol{W_m}$ are the data weighting matrix and regularization matrix respectively, and $\|\|^2$ is the squared $L2$ norm. The Lagrange multiplier λ is the very important parameter and controls the trade-off between model regularization and data misfit function. Constable et al. (1987) suggested a line search method with iteration whereas Loke and Barker (1996) started with the higher value which further decreases at subsequent iterations. Yi et al. (2003) used a spatially variable regularization parameter and calculated it using the spread functions and model parameter resolution matrix. Günther et al. (2006) chose the regularization term using L-curve criteria. In the present study, we considered a higher value of the regularization parameter with the damping factor of 0.5, which was reduced to a minimum of 0.01 after successive iteration for the ERT data and same strategy was used for the plane wave electromagnetic data.

The minimization of the objective function shown in Eq. (1) at the $(i+1)^{th}$ iteration in a Gauss-Newton strategy requires the solution for the model upgrade (Farquharson and Oldenburg, 2004):

$$\left(\boldsymbol{S}^T \boldsymbol{W}_d^T \boldsymbol{W}_d \boldsymbol{S} + \lambda \boldsymbol{W}_m^T \boldsymbol{W}_m\right)\Delta \boldsymbol{m}^{i+1} = \boldsymbol{S}^T \boldsymbol{W}_d^T \boldsymbol{W}_d \left(\boldsymbol{d}^{obs} - \boldsymbol{d}^i\right) - \lambda \boldsymbol{W}_m^T \boldsymbol{W}_m \boldsymbol{m}^i, \tag{3}$$

where, $\mathbf{S}$ is the Jacobian matrix and $\Delta\boldsymbol{m}$ is model perturbation vector. Since model parameters are transformed into logarithmic scale they are updated using the following expression:

$$\boldsymbol{m}^{i+1} = \boldsymbol{m}^i \exp\left(\Delta \boldsymbol{m}^{i+1}\right). \tag{4}$$

2.2. Fuzzy c means clustering

Here, the role of the clustering procedure is to divide each of the resistivity blocks into groups of similar geologic units. There are numerous clustering techniques available in the literature. Of these, Fuzzy C-Means (FCM) is a strategy for clustering which permits one piece of data to belong to two or more groups (Hoppner et al., 1999; Hathaway and Bezdek, 2001).

Let $\boldsymbol{m} = [m_1, m_2, \ldots, m_M]^T$ denote the M number of resistivity blocks to be partitioned into C number of geologic units (number of cluster centers). Assuming that we know the number of geologic units in the study area (i.e., the number of groups/clusters, C), Iterative minimization of a particular objective function could be performed. Further, we obtained the optimum value of the cluster centers and their respective membership values ($\boldsymbol{p}$). The FCM clustering could be solved by minimizing the following objective function (e.g., Bezdek, 1981):

$$\phi_c\left(p_{jk}, u_k\right) = \sum_{j=1}^{M} \sum_{k=1}^{C} p_{jk}^q \|m_j - u_k\|^2, \tag{5}$$

Subject to $0 \le p_{jk} \le 1$,

$$\sum_{k=1}^{C} p_{jk} = 1 \;\forall\; j \in \{1, 2, \ldots, \mathrm{M}\}, \tag{6}$$

where, C is the number of geologic units in the study area, M is the number model parameters, and p_{jk} indicates the degree of membership of the model parameter m_j to the kth cluster defined by its center u_k. Weighting exponent q is a fuzzification parameter, which is the degree of overlap between the geologic units (clusters). As q approaches infinity, the solution approaches its highest degree of fuzziness (e.g., Bezdek, 1981). We chose $q = 2$ because it is acknowledged as a decent decision of the fuzzification parameter by numerous scientists (e.g., Hathaway and Bezdek, 2001). If we take the derivative of Eq. (5) with respect to p_{jk} by assuming cluster centers u_k as a constant, and setting it to zero and then obtain the update of membership values p_{jk} (Hathaway et al., 2000; Sun and Li, 2015):

$$p_{jk} = \frac{1}{\sum_{i=1}^{C} \left(\frac{\|m_j - u_k\|}{\|m_j - u_i\|}\right)^{\frac{2}{q-1}}}, \tag{7}$$

Similarly, the differentiation of Eq. (5) with respect to u_k by assuming membership values p_{jk} as a constant, and setting it to zero