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An efficiently dynamic stress strain simulation method on digital rock

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ABSTRACT

The dynamic stress strain simulation is one of the several digital rock physics methods that calculate elastic property of digital rocks. Simulations for calculating P- and S-wave property of one digital rock includes one compressional simulation and three shear simulations. Moreover, the simulation frequency must be low enough to meet the long wavelength assumption. Hence, heavy computational burden is unavoidable. To improve the computational efficiency, in addition to parallel computation, one possible way is to optimize the boundary condition applied on the digital rocks. The idea is to integrate the four kinds of boundary condition which correspond to the four separated simulations in the existed method into one boundary condition, accordingly the number of simulation reduces to one. The computational efficiency of the new simulation method improves approximately four times, and the numerical accuracy is proved to be unaltered.

1. Introduction

The study of the elastic property of porous rocks is one of the important researching fields in exploration seismology and rock physics. In laboratory, low frequency stress strain measurement (Batzle et al., 2006; Tisato and Quintal, 2014; Pimienta et al., 2015, 2016; Spencer and Shine, 2016) is utilized to measure the elastic property of rock samples. In order to compare the measured results to those of seismic, sonic and ultrasonic wave measurements, levels of applied strain must be closed to or less than 10^{-6}.

Digital rock physics simulates physical fields on digital rocks, which are high-resolution representations of rock samples in computer, to obtain effective physical property of the entire digital rocks. It opens a door for calculation of the elastic property on the digital rocks in the 1990s (Garboczi and Day, 1995; Garboczi, 1998). There are mainly two kinds of simulation. One is the linear elastic static simulation, or usually called as finite element method (Arns et al., 2002; Dvorkin et al., 2011; Zhang et al., 2016), and the other one is the transmitted wave simulation (Saenger et al., 2007; Saenger et al., 2011; Saenger et al., 2016; Zhu and Shan, 2016). They are carried out to study relationships between effective elastic property and porosity, pore shape as well as pore fluids. An alternative way to calculate the elastic property, which corresponds to the stress strain measurement (Batzle et al., 2006) in reality, is the dynamic stress strain simulation on digital rocks developed by Zhang and Toksöz (2012). Using a digital rock of Berea sandstone built from computed tomography (CT) images, they compute the elastic property by both the dynamic simulation and linear elastic static simulation. Results of the two methods are closed to each other. This indicates that the dynamic simulation has the potential to be an accurate numerical analysis method. To calculate the elastic moduli of the digital rock, Zhang and Toksöz (2012) carry out four separate simulations with different boundary conditions, namely one pure compressional simulation and three pure shear simulations. Its efficiency is low especially when the size and the number of digital rocks are large.

In this article, we develop a dynamic stress strain simulation method that yields the elastic property by one numerical simulation. The main idea is to superimpose the four kinds of boundary condition used by Zhang and Toksöz (2012) separately into one integrated boundary condition and then solve the elastic wave equations by finite difference method. Finally, the elastic property is extracted from the time varied average stress curve and average strain curve of the entire digital rock.

2. Methodology

The digital rock is usually a rectangular or cubic model composed of tens of thousands of small cubic grids. It is naturally a model for numerical simulation as long as appropriate boundary condition and physical governing equation are applied. For dynamic stress strain simulations, we solve elastic wave equations under strain boundary condition which is displacement or velocity components are set on the surfaces of the digital rocks.

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2.1. Boundary condition

We assume in this work that the digital rock is cubic with its front and back surfaces perpendicular to *X*-axis, left and right surfaces perpendicular to *Y*-axis, and upper and lower surfaces perpendicular to *Z*-axis.

The four kinds of simulation of Zhang and Toksöz (2012) are shown in Fig. 1. Fig. 1a indicates the compressional simulation. The six surfaces of the digital rock are compressed by external forces perpendicular to the surfaces. In this case, the volume of the digital rock decreases, but its shape remains unchanged. For an effectively isotropic digital rock, the bulk modulus is calculated from this simulation. Fig. 1b, c and d represent three shear simulations. In each shear simulation, four of the digital rock's surfaces are dragged by external forces tangent to the surfaces and parallel with the other two surfaces. In the shear simulations, only the shape of the digital rock changes, the volume remains unchanged. Three shear moduli corresponding to the three simulations in Fig. 1b to d are calculated. For an effectively isotropic digital rock, its shear modulus is the average of the above three shear moduli.

Fig. 2 is a sketch map of the boundary condition of our simulation method. By comparing Fig. 1 with Fig. 2, we can find that the boundary condition in Fig. 2 is the superimposing of the four boundary conditions in Fig. 1. Under this boundary condition, both the volume and the shape of the digital rock vary in the simulation. The bulk and shear moduli are calculated from the output average stress and strain curves of the simulation.

Fig. 2 is only a schematic diagram of the boundary condition. To be more clearly, we express the boundary condition in formulas. As we known, it is easy to set the value of displacement or velocity of the grid points in the digital rock. We find in the finite element method of Garboczi (1998), the initial displacement field of the digital rock is defined by following formulas

$$\begin{aligned} u_x(l,m,n) &= l\cdot\varepsilon_{xx} + m\cdot\varepsilon_{xy} + n\cdot\varepsilon_{zx} \\ u_y(l,m,n) &= l\cdot\varepsilon_{xy} + m\cdot\varepsilon_{yy} + n\cdot\varepsilon_{yz} \\ u_z(l,m,n) &= l\cdot\varepsilon_{zx} + m\cdot\varepsilon_{yz} + n\cdot\varepsilon_{zz}. \end{aligned} \tag{1}$$

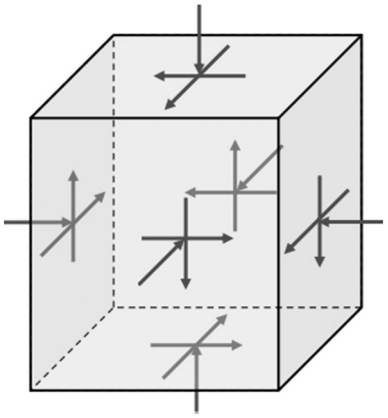

Fig. 2. Sketch map of the boundary condition of our simulation. Arrows represent the external forces acting on the digital rock.

where, u_x, u_y, u_z are the displacement components at every grid point along the *X*-, *Y*- and *Z*-axis directions; *l*, *m* and *n* are indexes of the grid points along the *X*-, *Y*- and *Z*-axis directions; ε_{xx}, ε_{yy}, ε_{zz}, ε_{xy}, ε_{yz}, ε_{zx} are strain components of the entire digital rock. According to the elasticity theory, there are nine strain components, but the shear strain components $\varepsilon_{xy}=\varepsilon_{yx}$, $\varepsilon_{yz}=\varepsilon_{zy}$, $\varepsilon_{zx}=\varepsilon_{xz}$. Hence the number of independent strain components is six. The displacement field determines the initial strain and stress fields of the digital rock and then the initial elastic potential energy. Garboczi (1998) uses the conjugate gradient method to reach the minimum energy status. The elastic moduli of the digital rock are inverted from the average stress and strain of the entire digital rock. The evolution of stress and strain fields on the digital rock during the conjugate gradient iteration is not a true physical process. Therefore, we compute the elastic moduli from the final stable elastic status.

Similarly, we define the displacement field on digital rock's surfaces, and the displacement field inside the digital rock is determined

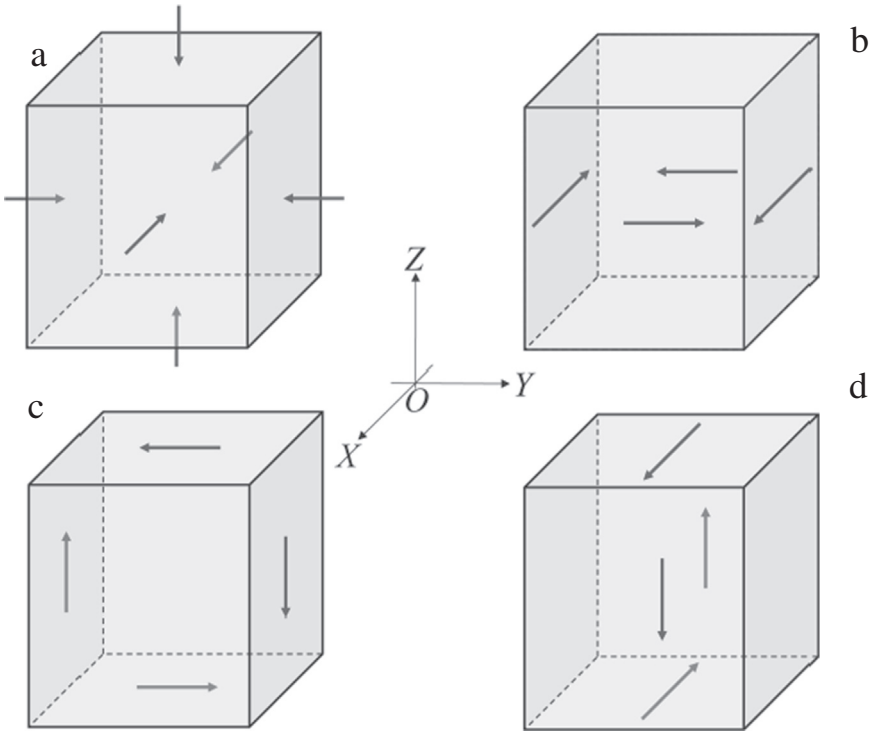

Fig. 1. Illustration of the original dynamic stress strain simulation method. Arrows represent the external forces acting on the digital rocks. The four kinds of simulations are: (a) compressional simulation; (b), (c) and (d) shear simulations.