Physics of the Earth and Planetary Interiors 277 (2018) 113–132

Contents lists available at ScienceDirect

Physics of the Earth and Planetary Interiors

journal homepage: www.elsevier.com/locate/pepi

Systematic parameter study of dynamo bifurcations in geodynamo simulations

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ARTICLE INFO

Keywords:
Earth core
Magnetohydrodynamics (MHD)
Dynamo: theory and simulations
Numerical solution
Planetary interiors

ABSTRACT

We investigate the nature of the dynamo bifurcation in a configuration applicable to the Earth's liquid outer core, i.e. in a rotating spherical shell with thermally driven motions with no-slip boundaries. Unlike in previous studies on dynamo bifurcations, the control parameters have been varied significantly in order to deduce general tendencies. Numerical studies on the stability domain of dipolar magnetic fields found a dichotomy between non-reversing dipole-dominated dynamos and the reversing non-dipole-dominated multipolar solutions. We show that, by considering weak initial fields, the above transition disappears and is replaced by a region of bistability for which dipolar and multipolar dynamos coexist. Such a result was also observed in models with free-slip boundaries in which the geostrophic zonal flow can develop and participate to the dynamo mechanism for non-dipolar fields. We show that a similar process develops in no-slip models when viscous effects are reduced sufficiently.

The following three regimes are distinguished: (i) Close to the onset of convection (Ra_c) with only the most critical convective mode (wave number) being present, dynamos set in supercritically in the Ekman number regime explored here and are dipole-dominated. Larger critical magnetic Reynolds numbers indicate that they are particularly inefficient. (ii) in the range $3 < Ra/Ra_c < Ra_c$, the bifurcations are subcritical and only dipole-dominated dynamos exist. (iii) in the turbulent regime ($Ra/Ra_c > 10$), the relative importance of zonal flows increases with Ra in non-magnetic models. The field topology depends on the magnitude of the initial magnetic field. The dipolar branch has a subcritical behavior whereas the multipolar branch has a supercritical behavior. By approaching more realistic parameters, the extension of this bistable regime increases. A hysteretic behavior questions the common interpretation for geomagnetic reversals.

Far above the dynamo threshold (by increasing the magnetic Prandtl number), Lorentz forces contribute to the first order force balance, as predicted for planetary dynamos. When Ra is sufficiently high, dipolar fields affect significantly the flow speed, the flow structure and heat transfer which is reduced by the Lorentz force regardless of the field strength. This physical regime seems to be relevant for studying geomagnetic processes.

1. Introduction

The mechanism whereby planets maintain magnetic fields against ohmic decay is one of the longest standing problems in science. It is commonly believed that the magnetic field is generated by electromotive forces driven by electrically conducting fluid motions in celestial bodies, namely dynamo action (Moffatt, 1978; Dormy and Soward, 2007). Dynamo action is considered to be responsible for the presence of magnetic activity for a large variety of astrophysical objects including planets, stars and galaxies. Planetary magnetic fields result from dynamo action thought to be driven by convection in electrically conducting fluid regions. Convection in these systems is strongly influenced by the Coriolis force resulting from global planetary rotation.

Since the time of the first fully three-dimensional numerical models (e.g. Glatzmaier and Roberts, 1995), there have been significant advances in our understanding of the fluid dynamics of planetary cores. Many features of the Earth's magnetic field have been reproduced numerically (Christensen et al., 1998; Christensen et al., 1999; Busse et al., 1998; Takahashi et al., 2005; Christensen and Wicht, 2007) even though realistic parameters differ by several orders of magnitude in direct numerical simulations. For instance, the Ekman number for the Earth's outer core is approximately $E = 10^{-15}$ whereas $E \geqslant 10^{-6}$ can be considered in numerical models (see below for a complete definition of

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https://doi.org/10.1016/j.pepi.2018.02.001
Received 24 May 2017; Received in revised form 24 January 2018; Accepted 5 February 2018
Available online 22 February 2018
0031-9201/

this dimensionless number). Possible field generation mechanisms in planetary conducting zones have been proposed by Olson et al. (1999) and dynamo coefficients have been calculated in geodynamo models (Schrinner et al., 2007, 2012) using the test-field method (Schrinner et al., 2005). Progress both in numerical methods as well as in parallel computer architecture has made it possible to explore an extensive parameters space in order to deduce the physical ingredients responsible for the dominance of the axial dipole field (Christensen and Aubert, 2006; King et al., 2010; Schrinner et al., 2012; Soderlund et al., 2012).

From numerical data and theoretical arguments, Christensen and Aubert (2006) have proposed scaling laws in order to predict observables as the magnitude of velocity field and magnetic field for planets (see also Stelzer and Jackson, 2013) and for rapidly rotating stars (Christensen et al., 2009). However, their predictive character and their relevance have been recently questioned by Oruba and Dormy (2014) and Tilgner (2014). In addition, according to several recent numerical studies (King et al., 2010; Soderlund et al., 2012; King and Buffett, 2013), viscosity could play an important role in numerical results. Numerical simulations often explore a physical regime in which viscous effects would dominate inertia whereas the opposite situation is believed to hold in planetary interiors. According to Davidson (2014), helical motions responsible for dynamo processes of dipolar morphology in simulations would result from the importance of viscosity. The relevance of numerical results to improve our knowledge of planetary dynamos is still a question of debate that we address in particular in this paper.

Recently, extreme runs have been carried out in order to reduce the effects of viscosity (Soderlund et al., 2015; Yadav et al., 2016; Schaeffer et al., 2017). In these runs, the flow is strongly affected by the Lorentz force which appears as one of the dominant forces, i.e. Lorentz and Coriolis forces would comprise the leading-order force balance (MAC balance). Aubert et al. (2017) have reached very low values of the large-scale viscosity by using a different numerical approach (large-eddy simulations) and they argue for a continuous path connecting today's simulations with planetary interiors. Aurnou and King (2017) argue that the influence of the Lorentz force depends on the scale and global scale force balance would be geostrophic. Convection would be influenced by the magnetic field only below a certain scale in simulations and in the Earth's outer core. In this paper, we show that the impact of the Lorentz force depends on the buoyant forcing in our dataset.

Observations and numerical simulations indicate that rapid global rotation and thus the ordering influence of the Coriolis force is of major importance for the generation of coherent magnetic fields (Stellmach and Hansen, 2004; Käpylä et al., 2009; Brown et al., 2010). Kutzner and Christensen (2002) demonstrated the existence of a dipolar and a multipolar dynamo regime and Christensen and Aubert (2006) showed that the transition between the two regimes is governed by a local Rossby number (Ro_ℓ), i.e. by the influence of inertia relative to the Coriolis force. Similar results were reported by Sreenivasan and Jones (2006), as well. Dipolar models were found for small Rossby numbers; they are separated by a fairly sharp regime boundary from multipolar models, where inertia is more important. The models transition from a dipolar morphology to a multipolar state as the local Rossby number increases above a certain value ($Ro_\ell > 0.1$). By considering different initial conditions for the magnetic field, we will show below that multipolar dynamos can be generated for local Rossby numbers lower than 0.1 even if no-slip boundaries are used.

Paleomagnetic measurements have allowed us to reconstruct the dynamics of the magnetic field. Irregularly over geologic time, the Earth's magnetic polarity has changed sign and such reversals have occured several hundred times during the past 160 million years. Glatzmaier and Roberts (1995) were the first to simulate such events numerically. Olson and Christensen (2006) have inferred some of the physical causes associated with field reversals in planetary interiors from numerical studies. Reversals would result from the importance of inertia relative to the Coriolis force. If the local Rossby number is close to the transitional value ($Ro_\ell \approx 0.1$), the dynamos are dipole-dominated and exhibit sporadic polarity reversals. In the light of our results, we will question the explanation proposed by Olson and Christensen (2006) for geomagnetic reversals.

By considering lower Ekman numbers and different magnetic Prandtl numbers, we extend the study by Morin and Dormy (2009). Decreasing the Ekman number allows one to explore smaller magnetic Prandtl numbers as highlighted by Christensen and Aubert (2006). It is of primary interest to understand dynamo bifurcations for very low E and Pm as these numbers are known to be 10^{-15} and 10^{-6} respectively, in the Earth's outer core and of similar order of magnitude in other planetary dynamo regions or in rapidly rotating stellar interiors.

In Section 2, we present the differential equations and input/output parameters. Section 3 is devoted to a hydrodynamic study and a kinematic study in which the dynamo threshold as a function of hydrodynamic forcing is determined for simple cases. A systematic study of the dynamo bifurcation is addressed in Section 4 and we discuss these results in Section 5. In Section 6, we focus on the action of the fields on the flow through the Lorentz force. We also discuss the physical regimes which can give rise to dipolar dynamos. We conclude and apply our results to planetary magnetism in Section 7.

2. Equations and dimensionless parameters

Our dynamo models are solutions of the MHD-equations for a conducting Boussinesq fluid in a rotating spherical shell. The fluid motion is driven by convection due to an imposed temperature difference, ΔT (where T denotes the temperature), between the inner and the outer shell boundaries. The fundamental length scale of our models is the shell width L, we scale time by L^2/ν, with ν the kinematic viscosity, and temperature is scaled by ΔT and the magnetic field is considered in units of $\sqrt{\varrho\mu\eta\Omega}$, with ϱ denoting the density, μ the magnetic permeability, η the magnetic diffusivity and Ω the rotation rate. With these units, the dimensionless momentum, temperature and induction equations are

$$E\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} - \nabla^2\mathbf{v}\right) + 2\mathbf{z}\times\mathbf{v} + \nabla P = Ra\frac{\mathbf{r}}{r_o}T + \frac{1}{Pm}(\nabla\times\mathbf{B})\times\mathbf{B}, \quad (1)$$

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T = \frac{1}{Pr}\nabla^2 T, \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \frac{1}{Pm}\nabla^2\mathbf{B}. \quad (3)$$

Here, the unit vector $\vec{z}$ indicates the direction of the rotation axis. We also note that the velocity field $\vec{v}$ and the magnetic field $\vec{B}$ are solenoidal. The system of equations is governed by four dimensionless parameters, the Ekman number $E = \nu/\Omega L^2$, the (modified) Rayleigh number $Ra = \alpha_T g_0 \Delta T L/\nu\Omega$, the Prandtl number $Pr = \nu/\kappa$, and the magnetic Prandtl number $Pm = \nu/\eta$. In these definitions, α_T stands for the thermal expansion coefficient, g_o is the gravitational acceleration at the outer boundary, and κ is the thermal diffusivity. Another control parameter is the aspect ratio of the shell defined as the ratio of the inner to the outer shell radius, $\chi = r_i/r_o$. It determines the width of the convection zone and is fixed in our study at 0.35. The mechanical boundary conditions are no-slip at both boundaries. Furthermore, the magnetic field matches a potential field outside the fluid shell and fixed temperatures are prescribed at both boundaries.

Some of the models investigated here exhibit bistability where the solution depends on the initial conditions for the magnetic field. A strong ($\Lambda \approx 10$) initial dipolar field gives rise to a dipolar solution whereas a multipolar solution is obtained when a weak seed field is considered as an initial condition (see Section 4). Some calculations were started from a numerical solution with slightly different