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Effects of radial distribution of entropy diffusivity on critical modes of anelastic thermal convection in rotating spherical shells

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ABSTRACT

Linear stability analysis of anelastic thermal convection in a rotating spherical shell with entropy diffusivities varying in the radial direction is performed. The structures of critical convection are obtained in the cases of four different radial distributions of entropy diffusivity; (1) κ is constant, (2) κT_0 is constant, (3) $\kappa\rho_0$ is constant, and (4) $\kappa\rho_0 T_0$ is constant, where κ is the entropy diffusivity, T_0 is the temperature of basic state, and ρ_0 is the density of basic state, respectively. The ratio of inner and outer radii, the Prandtl number, the polytropic index, and the density ratio are 0.35, 1, 2, and 5, respectively. The value of the Ekman number is 10^{-3} or 10^{-5}. In the case of (1), where the setup is same as that of the anelastic dynamo benchmark (Jones et al., 2011), the structure of critical convection is concentrated near the outer boundary of the spherical shell around the equator. However, in the cases of (2), (3) and (4), the convection columns attach the inner boundary of the spherical shell.

A rapidly rotating annulus model for anelastic systems is developed by assuming that convection structure is uniform in the axial direction taking into account the strong effect of Coriolis force. The annulus model well explains the characteristics of critical convection obtained numerically, such as critical azimuthal wavenumber, frequency, Rayleigh number, and the cylindrically radial location of convection columns.

The radial distribution of entropy diffusivity, or more generally, diffusion properties in the entropy equation, is important for convection structure, because it determines the distribution of radial basic entropy gradient which is crucial for location of convection columns.

1. Introduction

The problem of thermal convection in rotating spherical shells has been investigated vigorously since middle of 20th century as an application to fluid motions in the interiors of stars, gas and icy planets (e.g. Chandrasekhar, 1961). While most of the studies assume the Boussinesq approximation, researches using the anelastic approximation have become active recently.

In contrast to Boussinesq approximation where basic density and material properties of the fluid are constant, there are varieties of radial distributions of thermodynamic variables and properties in anelastic approximation. Most of fundamental studies on this topics from the viewpoint of geophysical and astrophysical fluid dynamics assume ideal gas, polytropic relation and hydrostatic balance (e.g. Jones et al., 2009, 2011). Numerical simulations of the solar convection zone apply radial distributions of density, temperature and pressure derived from helio-seismology and/or solar evolution models as a basic state (e.g. Brun et al., 2004, Browning, 2008). However, there is no guiding principle for determining viscosity and thermal diffusivity distributions. Molecular viscosity and thermal diffusivity in gas planet atmospheres are estimated by first principle calculations based on the molecular dynamics (e.g. French et al., 2012). However, the molecular diffusivities may be inappropriate for global thermal convection models, since small scale fluid motions governed by the molecular diffusivities could not be resolved in the present global convection model due to the restriction of numerical resources. From this standpoint, eddy entropy diffusion is selected as a thermal diffusive process in many studies.

The radial distributions of viscosity and entropy diffusivity must influence on the radial location of convection. Especially, entropy diffusivity is important since its radial distribution directly affects the diffusive entropy distribution, whose radial gradient

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determines the local static stability. However, most of geophysical and astrophysical fluid dynamic studies assume constant kinematic viscosity and entropy diffusivity for simplicity of the formulation (e.g. Jones et al., 2009, 2011), and their results show that convective columns concentrate near the outer boundary around the equator when the density contrast is sufficiently large. Note that this setup is also the only one which ensures that the control parameters, such as the Ekman, Prandtl, Rayleigh numbers, do not depend on radius.

Some simulations of the solar convection zone assume that viscosity and diffusivity is in proportion to the $-1/2$ power of density (e.g. Brun et al., 2004; Featherstone and Miesch, 2015). A pioneering study by Glatzmaier and Gilman (1981) investigates the effects of radial distributions of viscosity and entropy (potential temperature) diffusivity at the same time under a moderate rotation rate, and shows that location of convection moves from the outer to inner regions as the diffusivities are increased in the outer regions and are decreased in the inner regions. However, it is difficult to determine which diffusivity governs the location of convection from their results.

In this paper, we investigate structure of critical thermal convection in anelastic fluids in a rotating spherical shell, with entropy diffusivities varying in the radial direction, and show that the radial distribution of the diffusivities is crucial to the location of the convection. In Section 2, the anelastic model and formulation of linear stability analyses are described. The radial distributions of entropy diffusivity used in this study are also mentioned. In Section 3, we illustrate the location of convection drastically changes depending on the distribution of diffusivity. In Section 4, we develop a rapidly rotating two-dimensional annulus model for anelastic systems by extending the annulus model for Boussinesq systems proposed by Busse (1986), Busse and Or (1986) and Busse and Simitev (2014) as a simplified conceptual model for three-dimensional rapidly rotating spherical convection systems. Note that the annulus model developed here does not aim at an approximation of the full spherical model with a high accuracy. We would like to obtain and express essential physical mechanisms of anelastic convection in rotating spheres and spherical shells by using the annulus model. Section 5 summarizes the results.

2. Model

2.1. Background state of the anelastic system

We consider thermal convection of ideal gas in a rotating spherical shell. The background state of the anelastic system satisfies polytropic relation and hydrostatic balance. Gravity is determined by the mass of the inner sphere, and self gravitational force is not taken into consideration. Background density ρ_0 and pressure p_0 non-dimensionalized with the values at the middle of the spherical shell are expressed as follows (Jones et al., 2009):

$$\rho_0 = T_0^n, \quad p_0 = T_0^{n+1}, \quad T_0 = c_0 + \frac{c_1}{r}, \tag{1}$$

$$c_0 = \frac{2\theta_o - \chi - 1}{1-\chi}, \quad c_1 = \frac{(1+\chi)(1-\theta_o)}{(1-\chi)^2}, \tag{2}$$

$$\theta_o = \frac{\chi + 1}{\chi \exp(N_\rho/n) + 1}, \quad \theta_i = \frac{1 + \chi - \theta_o}{\chi}. \tag{3}$$

Here, r is the radial coordinate, r_i and r_o are the inner and outer radii of the spherical boundary, respectively, $\chi = r_i/r_o$ is the radius ratio of the shell, $N_\rho = \ln[\rho(r_i)/\rho(r_o)]$ is the logarithm of density ratio between the inner and outer spheres, and n is the polytropic index. For a perfect gas, heat capacity ratio γ should be related to polytropic index as $\gamma = 1 + 1/n$ to ensure adiabatic background state (e.g. Jones et al., 2011). We fix these parameters as $\chi = 0.35$ ($r_i = 0.538, r_0 = 1.538$), $N_\rho = 5$, and $n = 2$ (Fig. 1).

2.2. Governing equations for disturbances

We investigate linear stability of the basic state in thermal conductive equilibrium state with no fluid motion. The radial entropy distribution of the basic state is,

$$\frac{d\bar{S}}{dr} = -\frac{C}{\kappa \rho_0 T_0 r^2}, \quad C = \left(\int_{r_i}^{r_o} \frac{1}{\kappa \rho_0 T_0 r^2} dr \right)^{-1}, \tag{4}$$

where $\kappa(r)$ is entropy diffusivity normalized with the value at the middle of the shell. The linearized equations of disturbances with respect to this basic state are,

$$\nabla \cdot (\rho_0 \boldsymbol{u}) = 0, \tag{5}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{2}{\mathrm{E}} \boldsymbol{k} \times \boldsymbol{u} = -\nabla \left(\frac{p}{\rho_0} \right) + \frac{\mathrm{Ra}}{\mathrm{Pr}} \frac{S}{r^2} \boldsymbol{e}_r + \frac{1}{\rho_0} \boldsymbol{F}_v,$$

$$(\boldsymbol{F}_v)_i = \frac{\partial}{\partial x_j} \left\{ \rho_0 \nu(r) \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) \right\}, \tag{6}$$

$$\frac{\partial S}{\partial t} + \frac{d\bar{S}}{dr} u_r = \frac{1}{\mathrm{Pr}} \frac{1}{\rho_0 T_0} \nabla \cdot (\rho_0 T_0 \kappa(r) \nabla S). \tag{7}$$

$\boldsymbol{u}, S, p$ are velocity, entropy disturbance and pressure disturbance, respectively. r and $\boldsymbol{e}_r$ are radial coordinate and the unit vector in the radial direction, respectively. Nondimensional parameters appearing in the governing equations are the Rayleigh number Ra, the Ekman number E, the Prandtl number Pr, which are defined as follows:

$$\mathrm{Ra} = \frac{g_0 \Delta S D^3}{\kappa_c \nu_c C_p}, \quad \mathrm{E} = \frac{\nu_c}{\Omega D^2}, \quad \mathrm{Pr} = \frac{\nu_c}{\kappa_c}. \tag{8}$$

here, g_0 is the value of gravitational acceleration at $r = 1$, ΔS is entropy difference between inner and outer spheres, C_p is specific heat capacity at constant pressure, $D = r_o - r_i$ is the thickness of the shell, r_i, r_o are the radius of the inner and outer sphere, κ_c, ν_c are the values of entropy diffusivity and kinematic viscosity at the middle of the sphere. In addition to these non-dimensional parameters, the radius ratio χ appears in the boundary conditions. We

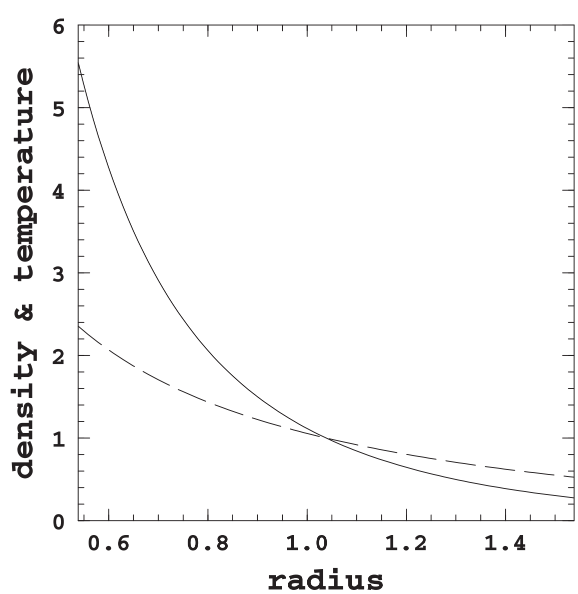

Fig. 1. Radial distributions of the basic state. The solid line is density, broken line is temperature. $\chi = 0.35, N_\rho = 5$, and $n = 2$. The horizontal axis covers the whole shell in the radial direction ($r_i = 0.538, r_0 = 1.538$).

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