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Effects of anisotropic turbulent thermal diffusion on spherical magnetoconvection in the Earth's core

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ABSTRACT

We re-consider the plate-like model of turbulence in the Earth's core, proposed by Braginsky and Meytlis (1990), and show that it is plausible for core parameters not only in polar regions but extends to mid- and low-latitudes where rotation and gravity are not parallel, except in a very thin equatorial layer. In this model the turbulence is highly anisotropic with preferred directions imposed by the Earth's rotation and the magnetic field. Current geodynamo computations effectively model sub-grid scale turbulence by using isotropic viscous and thermal diffusion values significantly greater than the molecular values of the Earth's core. We consider a local turbulent dynamo model for the Earth's core in which the mean magnetic field, velocity and temperature satisfy the Boussinesq induction, momentum and heat equations with an isotropic turbulent Ekman number and Roberts number. The anisotropy is modelled only in the thermal diffusion tensor with the Earth's rotation and magnetic field as preferred directions. Nonlocal organising effects of gravity and rotation (but not aspect ratio in the Earth's core) such as an inverse cascade and nonlocal transport are assumed to occur at longer length scales, which computations may accurately capture with sufficient resolution. To investigate the implications of this anisotropy for the proposed turbulent dynamo model we investigate the linear instability of turbulent magnetoconvection on length scales longer than the background turbulence in a rotating sphere with electrically insulating exterior for no-slip and isothermal boundary conditions. The equations are linearised about an axisymmetric basic state with a conductive temperature, azimuthal magnetic field and differential rotation. The basic state temperature is a function of the anisotropy and the spherical radius. Elsasser numbers in the range 1–20 and turbulent Roberts numbers 0.01–1 are considered for both equatorial symmetries of the magnetic basic state. It is found that anisotropic turbulent thermal diffusivity has a strong destabilising effect on magneto-convective instabilities, which may relax the tight energy budget constraining geodynamo models. The enhanced instability is not due to a reduction of the total diffusivity. The anisotropy also strengthens instabilities which break the symmetry of the underlying state, which may facilitate magnetic field reversal. Geostrophic flow appears to suppress the symmetry breaking modes and magnetic instabilities. Through symmetry breaking and the geostrophic flow the anisotropy may provide a mechanism of magnetic field reversal and its suppression in computational dynamo models.

1. Introduction

Geodynamo simulations cannot achieve Earth core parameter values, see for example Schaeffer et al. (2017). Approaches to model sub-grid scale turbulence in computations for the Earth's liquid core include filtering techniques, scaling arguments and parametrised models; see for example Braginsky and Roberts (1995), Phillips and Ivers (2000), Matsushima (2001), Buffett (2003), Matsui and Buffett (2005), Moffatt (2008), Calkins et al. (2015), Aurnou et al. (2015), Aubert et al. (2017).

Turbulence modelling requires heuristic and inductive assumptions of varying plausibility, which are not rigorously deducible from existing theory. Observational data or results of laboratory experiments or computations at realistic values of core parameters are not available as in atmospheric or oceanic turbulence, or in engineering applications. We adopt the hypothesis of Braginsky

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and Meytlis (1990) that core turbulence is anisotropic with the thermal and viscous diffusivities of the mean temperature and velocity enhanced in preferred directions to values comparable to the molecular magnetic diffusivity (see Phillips and Ivers, 2000). Davidson and Siso-Nadal (2002) found support for this picture in the destruction of angular momentum in core turbulence. Localized numerical simulations (St Pierre, 1996; Matsushima et al., 1999; Matsui and Buffett, 2005), which find that the turbulent diffusion is enhanced parallel to the rotation axis and the toroidal magnetic field, also give support to this picture. Matsushima et al. (1999) corrected the dispersion relation of Braginsky and Meytlis (1990, Eq. (5.1)) for lower latitudes. We verify this and consider its implications for turbulence with Earth-like values.

Anisotropic thermal and viscous diffusion in a rapidly rotating electrically conducting fluid have been studied previously. The thermal and viscous diffusion are modelled by $\nabla\cdot(\boldsymbol{\kappa}\cdot\nabla\overline{\Theta})$ and $\nabla\cdot(\boldsymbol{\nu}\cdot\nabla\overline{\mathbf{v}})$, where $\boldsymbol{\kappa}$ and $\boldsymbol{\nu}$ are the turbulent thermal and viscous diffusion tensors respectively, and $\overline{\Theta}$ and $\overline{\mathbf{v}}$ are the mean temperature and velocity. More complicated viscous diffusion models are possible in principle, where $\boldsymbol{\nu}$ is a rank-4 tensor, $\nabla\cdot(\boldsymbol{\nu}\cdot\cdot\nabla\overline{\mathbf{v}})$, or includes the magnetic field $\nabla\cdot(\boldsymbol{\nu}\cdot\cdot\nabla\overline{\mathbf{v}}+\boldsymbol{\nu}_{\mathrm{m}}\cdot\cdot\nabla\overline{\mathbf{B}})$, where $\overline{\mathbf{B}}$ is the mean magnetic field and $\boldsymbol{\nu}_{\mathrm{m}}$ is also a rank-4 tensor, and $[\boldsymbol{\nu}\cdot\cdot\nabla\overline{\mathbf{v}}]_{ij}=\nu_{ijkl}\overline{v}_{k,l}$, etc in Cartesian tensor form. However, even the simpler model is computationally extremely difficult. Brestensky et al. (2005) consider the linear stability of a horizontal plane layer rapidly rotating about an axis perpendicular to the layer with vertical gravity, an imposed azimuthal magnetic field about the rotation axis and anisotropic thermal diffusion of the same form as considered herein. The boundaries are perfectly conducting and stress free. Modal expansions are possible for the solutions, which strongly decouple the magnetic induction, momentum and heat equations to yield a set of ordinary differential equations in the vertical coordinate. This allows high resolution calculations. The work has been extended in Brestensky et al. (2006) to include anisotropic viscous diffusion proportional to the anisotropic thermal diffusion in non-rotating and rotating thermal convection, and magnetoconvection in an applied uniform vertical magnetic field. Soltis et al. (2006) consider the effects of such anisotropic viscous and thermal diffusion on magnetoconvection in an applied uniform vertical magnetic field and their implications for torsional oscillations in the Earth's core. In both papers it is assumed that $\kappa_{ss}/\kappa_{zz}=\nu_{ss}/\nu_{zz}$, where $\kappa_{ss}=\mathbf{1}_s\cdot\boldsymbol{\kappa}\cdot\mathbf{1}_s, \nu_{ss}=\mathbf{1}_s\cdot\boldsymbol{\nu}\cdot\mathbf{1}_s$, are components of $\boldsymbol{\kappa}$ and $\boldsymbol{\nu}$, and (s,ϕ,z) are cylindrical polar coordinates with unit vectors $\mathbf{1}_s$, etc. Particularly noteworthy is the result of Brestensky et al. (2006) that if $\kappa_{ss}/\kappa_{zz}=\nu_{ss}/\nu_{zz}\ll 1$, then the inhibiting role of the magnetic field is suppressed, convection enhanced and the horizontal length scale is reduced, i.e. that an increase of the thermal and viscous diffusion parallel to the rotation axis compared to the transverse direction is de-stabilizing. Soltis and Brestensky (2010) consider stationary rotating magnetoconvection in an applied uniform horizontal magnetic field. They assume $\Delta:=\kappa_{xx}/\kappa_{zz}=\nu_{xx}/\nu_{zz}, \kappa_{yy}=\kappa_{zz}$ and $\nu_{yy}=\nu_{zz}$, and find that both the Braginsky-Meytlis anisotropy ($\Delta<1$) and anisotropy due to stratification ($\Delta>1$), as in a stratified layer at the top of the core, facilitate convection and shorten horizontal width of rolls, but in the stratification case the rolls are more perpendicular to the magnetic field than the rolls in the Braginsky-Meytlis case. Soltis and Brestensky (2013) consider magnetoconvection in a plane layer rotating about a horizontal axis in an applied uniform horizontal magnetic field, valid near the equator. Donald and Roberts (2004) considered the effects of anisotropic thermal diffusion on an intermediate dynamo model driven by buoyancy. They found that isotropy produced stronger magnetic field and thermal wind, and suggested that anisotropy of order $\kappa_{ss}/\kappa_{zz}\sim 0.4$ would more easily maintain the magnetic field.

Core turbulence driven by compositional buoyancy has been investigated by Loper and Moffatt (1993), Moffatt and Loper (1994), Loper et al. (2003) and Chulliat et al. (2004). Moffatt (2008) considered this type of core turbulence in the magnetostrophic approximation $Ro=0$ and $E=0$ at length-scales where an α-effect regenerates the magnetic field. Nonlinear effects are retained only through the advection of heat (Friedlander and Vicol, 2011, Friedlander et al., 2013). Calkins et al. (2015) developed a quasi-geostrophic dynamo model using a multi-scale analysis which exhibits anisotropy due to widely differing length scales parallel and perpendicular to the rotation axis. In these models the turbulence is necessary for magnetic field generation unlike the model we describe in Section 2.2.

In the present work we model the turbulent effects of thermal advection by anisotropic turbulent thermal diffusivity enhanced or diminished parallel to the rotation axis and the mean magnetic field. Following Roberts and Glatzmaier (2000, p. 1118), we assume that, since the Rossby number (defined below) $Ro\ll 1$, turbulent momentum transport is not significant in the core, except in boundary layers and possibly near the tangent cylinder. Hence anisotropic viscous effects are confined to boundaries and the viscous diffusivity is only isotropically enhanced in the fluid interior. This assumption has some support in computational models, e.g. Glatzmaier and Roberts (1995) and the Kuang and Bloxham (1997). The turbulent viscosity still orders of magnitude larger than molecular viscosity and ideally anisotropic viscous diffusion should be incorporated in the model. But we expect the effects of anisotropic heat transport in the core to be more significant and consider anisotropic thermal diffusion in isolation first. We assume that magnetic field generation occurs on resolvable length scales so that no α-effect occurs on the length scales of the turbulence.

The core V is assumed to be well-mixed due to the turbulence so that it is isentropic. The dimensional gravitational acceleration is $\mathbf{g}=-g_c\mathbf{r}/c$, where g_c is the gravitational acceleration at the core surface, $\mathbf{r}$ is the position vector. The length, time, speed, magnetic field and temperature are non-dimensionalised using the core radius c, the magnetic diffusion time $t_\eta:=c^2/\eta$ since we are interested in dynamo action, the magnetic diffusion speed $\mathcal{V}=\eta/c$, a typical magnetic field value $\mathcal{B}=\sqrt{\mu_0\rho 2\Omega\eta}$ and $\beta_\star c$, where $\beta_\star$ is a typical temperature gradient, μ_0 is the permeability of free space, η is the magnetic diffusivity, ρ is the fluid density. The effect of the centripetal acceleration is neglected. In the Boussinesq approximation the non-dimensionalised equations of momentum, magnetic induction, heat, mass conservation and Gauss's Law governing the magnetic field $\mathbf{B}$, the velocity $\mathbf{v}$ and the non-adiabatic temperature Θ in the boundary frame rotating with uniform angular velocity $\boldsymbol{\Omega}$ are

$$Ro\left(\partial_t\mathbf{v}+\mathbf{v}\cdot\nabla\mathbf{v}\right)+\mathbf{1}_z\times\mathbf{v}=-\nabla p+\mathbf{J}\times\mathbf{B}+R\Theta\mathbf{r}+E\nabla^2\mathbf{v} \tag{1.1}$$

$$\partial_t\mathbf{B}=\nabla^2\mathbf{B}+\nabla\times(\mathbf{v}\times\mathbf{B}) \tag{1.2}$$

$$\partial_t\Theta+\mathbf{v}\cdot\nabla\Theta=q\nabla^2\Theta+Q \tag{1.3}$$

$$\nabla\cdot\mathbf{v}=0,\quad \nabla\cdot\mathbf{B}=0, \tag{1.4}$$

where ∂_t is the derivative with respect to time t, $\mathbf{1}_z$ is the unit vector in the $\boldsymbol{\Omega}$ direction, $\mathbf{J}=\nabla\times\mathbf{B}$, the Rossby number $Ro=E_\eta:=\eta/2\Omega c^2$ where E_η is the magnetic Ekman number, $R:=\alpha_\Theta\beta_\star g_c c^2/2\Omega\eta$ is a modified Rayleigh number, $E:=\nu/2\Omega c^2$ is the Ekman number, $q:=\kappa/\eta$ is the Roberts number, κ is the molecular thermal diffusivity, α_Θ is the thermal expansivity, and Q is the heating rate per unit volume. The ratio $E/E_\eta=\nu/\eta$ is the magnetic Prandtl number. The magnetic Reynolds number $R_\eta:=\mathcal{V}c/\eta=1$. An aposteriori check on the scaling is that $\mathbf{v}$ is $\mathcal{O}(1)$ and $\mathbf{B}$ is $\mathcal{O}(1)$. The magnetic field is matched to an insulating exterior $\widehat{V}$, and the flow and temperature satisfy no-slip isothermal conditions on the core-mantle