

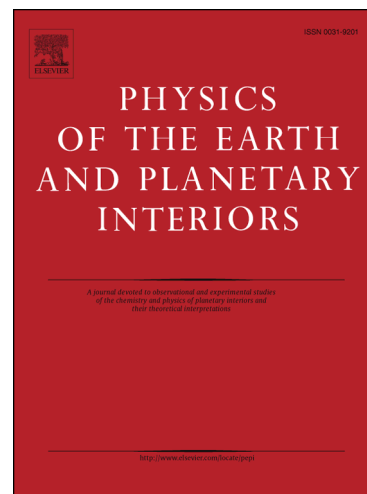
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Numerical solution of a non-linear conservation law applicable to the interior dynamics of partially molten planets

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13 **Abstract**

14 The energy balance of a partially molten rocky planet can be expressed as a non-  
15 linear diffusion equation using mixing length theory to quantify heat transport  
16 by both convection and mixing of the melt and solid phases. Crucially, in this  
17 formulation the effective or eddy diffusivity depends on the entropy gradient,  
18  $\partial S/\partial r$ , as well as entropy itself. First we present a simplified model with semi-  
19 analytical solutions that highlights the large dynamic range of  $\partial S/\partial r$ —around  
20 12 orders of magnitude—for physically-relevant parameters. It also elucidates  
21 the thermal structure of a magma ocean during the earliest stage of crystal  
22 formation. This motivates the development of a simple yet stable numerical  
23 scheme able to capture the large dynamic range of  $\partial S/\partial r$  and hence provide a  
24 flexible and robust method for time-integrating the energy equation.

25 Using insight gained from the simplified model, we consider a full model,  
26 which includes energy fluxes associated with convection, mixing, gravitational  
27 separation, and conduction that all depend on the thermophysical properties of  
28 the melt and solid phases. This model is discretised and evolved by applying the  
29 finite volume method (FVM), allowing for extended precision calculations and  
30 using  $\partial S/\partial r$  as the solution variable. The FVM is well-suited to this problem

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