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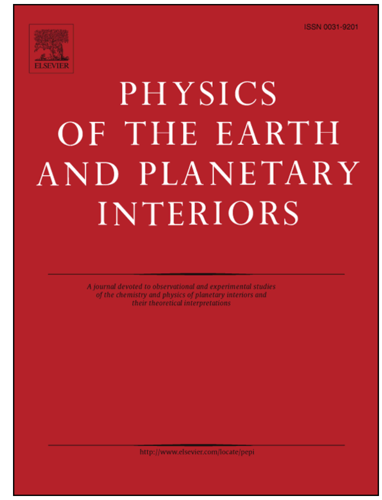

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Numerical solution of a non-linear conservation law applicable to the interior dynamics of partially molten planets

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Abstract

The energy balance of a partially molten rocky planet can be expressed as a non-linear diffusion equation using mixing length theory to quantify heat transport by both convection and mixing of the melt and solid phases. Crucially, in this formulation the effective or eddy diffusivity depends on the entropy gradient, $\partial S/\partial r$, as well as entropy itself. First we present a simplified model with semi-analytical solutions that highlights the large dynamic range of $\partial S/\partial r$—around 12 orders of magnitude—for physically-relevant parameters. It also elucidates the thermal structure of a magma ocean during the earliest stage of crystal formation. This motivates the development of a simple yet stable numerical scheme able to capture the large dynamic range of $\partial S/\partial r$ and hence provide a flexible and robust method for time-integrating the energy equation.

Using insight gained from the simplified model, we consider a full model, which includes energy fluxes associated with convection, mixing, gravitational separation, and conduction that all depend on the thermophysical properties of the melt and solid phases. This model is discretised and evolved by applying the finite volume method (FVM), allowing for extended precision calculations and using $\partial S/\partial r$ as the solution variable. The FVM is well-suited to this problem

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