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Efficient methodology for probabilistic analysis of consolidation considering spatial variability

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A B S T R A C T

The inherent spatial variability of natural soil deposits should be considered to obtain realistic probabilistically-based estimates of the time for soil consolidation. Although a variety of commercial software packages for solving consolidation analyses within numerical framework exist, they cannot incorporate spatial variability in a robust manner. Therefore, the consideration of the spatial variability is difficult and time consuming. This study proposes two efficient approaches for probabilistic analysis of consolidation considering the spatial variability of coefficient of consolidation (c), as well as a technique for calculating a representative c for combined vertical and radial consolidation. The proposed approaches, which include a first-order reliability (FORM) and stochastic response surface method (SRSM), were applied to examples of 1-D vertical consolidation and combined vertical and radial consolidation problems, and the applicability of the models and spatial variability are identified. As a result, the random field and numerical consolidation analysis were decoupled, and the proposed approaches were able to perform probabilistic consolidation analysis considering spatial variability using existing deterministic finite element codes without any modification. The probability of under-consolidation can be predicted by only 6 and 11 consolidation analysis using the FORM and SRSM approaches, respectively, and the accuracy of these two approaches was shown to be identical to the results of more computationally-expensive, conventional Monte Carlo simulation (MCS) results. It was found that the probabilistic consolidation analyses are sensitive to the spatial variability of c, and the variability of average degree of consolidation increases as the scale of fluctuation increases, which can be effectively identified and treated using either of the proposed approaches.

1. Introduction

The spatial variability of a given soil stratum may be described succinctly in terms of the three possible sources (Stuedlein and Bong, 2017): (1) the thickness of a given stratum, (2) the azimuthal extent of a given stratum, (3) the inherent variability within the given stratum, and/or a (4) combination of two or all of these sources. The spatial variability of soil stratum thickness, extent, and relevant engineering properties manifests in unavoidable uncertainty with regard to design, and may lead to an unexpected system response (Lacasse and Nadim, 1996). Probabilistic analyses considering the spatial variability of soil properties thus leads to a more rational framework for interpreting risk to project outcomes and schedule, the latter of which is commonly controlled by geotechnical concerns when soil consolidation comprises a significant challenge in a given project. Probabilistic analyses that incorporate the spatial variability of soil properties as random fields (RFs) are more appropriate than those considering soil properties as a single random variable that is invariant in space.

Numerous studies have been conducted to determine typical random field model (RFM) parameters (e.g., Phoon and Kulhawy, 1999; Fenton, 1999; Uzielli et al., 2005; Stuedlein et al., 2012a; Cao and Wang, 2012; Bong and Stuedlein, 2017; Wang et al., 2017) and study their influence in geotechnical problems that consider the spatial variability of soil properties over the last two decades (e.g., Fenton and Griffiths, 2001; Popescu et al., 2005; Cho, 2007; Srivastava et al., 2010; Griffiths et al., 2009; Stuedlein et al., 2012b; Chen et al., 2016; Bong and Stuedlein, 2017; Bong and Stuedlein, 2018). One of the most common geotechnical design problems, construction on soft ground and assessment of the duration of soil consolidation (e.g. Feng et al., 2001; Moo-Young et al., 2003; Cai et al., 2017), is also subject to the greatest uncertainties. Although significant uncertainty arises from drilling, sampling, and laboratory testing procedures (Ladd and DeGroot, 2003), considerable project risk arises from the inherent spatial variability of the compressible soil. However, few studies have been carried out to investigate the effects of spatial variability on soil consolidation. Badaoui et al. (2007) performed vertical consolidation analysis

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considering the spatial variability of elastic modulus and soil permeability, whereas Huang et al. (2008) investigated the influence of a spatially random coefficient of consolidation on one-dimensional consolidation. Bari et al. (2012) investigated the effect of spatial variability of soil permeability and volume compressibility on consolidation of soft soil by prefabricated vertical drains, but treated the spatially variability in axisymmetric geometry, which does not replicate the three-dimensional variability of actual soil deposits. Bong et al. (2014) analyzed consolidation by vertical drains considering the spatial variability of the coefficients of consolidation, and applied the stochastic response surface method (SRSM) for efficient probabilistic uncertainty propagation.

One of the main challenges in coupling spatial variability treated with random fields with consolidation analyses rests in the need to use numerical solutions to consolidation. Although a variety of commercial software packages that employ numerical solutions to solve consolidation analyses exist, they cannot incorporate the spatial variability in a robust manner. Furthermore, generation of non-proprietary analytical tools that can model RFs for consolidation analyses within a Monte Carlo Simulation (MCS) framework is time consuming and inefficient except for the most challenging projects. Thus, there is a need to develop robust and efficient analytical tools that are simple to implement in routine design analyses. Although various techniques such as importance sampling (Styblinski, 1979; Glynn, 1996), latin hypercube sampling (McKay et al., 1979), and subset simulation (Wang et al., 2011) have been proposed to improve the efficiency of the standard MCS, these approaches still require a large number of simulations, and if an existing code or commercial program does not support MCS, it is rather difficult to perform MCS. Alternative methods have been developed to address the computational expense associated with sophisticated MCS analyses including the response surface method (RSM, Box and Draper, 1987) and artificial neural networks (e.g., Goh and Kulhawy, 2005). Isukapalli et al. (1998) proposed the stochastic response surface method (SRSM) to extend the classic response surface method from deterministic space and this extension has been widely used to approximate full probabilistic analyses. For efficient slope reliability analysis, Jiang et al. (2015) proposed MCS-based system reliability analysis using representative slip surfaces and multiple stochastic response surfaces. Jiang et al. (2014) and Li et al. (2015) proposed non-intrusive stochastic finite element method and multiple response-surface method for slope reliability analysis considering spatial variability of soil properties. Huang et al. (2007) further extended SRSM to solve probabilistic finite element analyses with RFs generated using Karhunen-Loeve expansion (KLE). However, this method requires a significant number of collocation points for the generation of the response surface function to reduce the error. Additionally, the approach is general and does not directly treat consolidation analyses.

This paper proposes new and efficient probabilistic analytical approaches for vertical, radial, and combined vertical and radial consolidation considering the spatial variability in the vertical and radial coefficients of consolidation, c_v and c_r, respectively. Since the progress of consolidation is governed by the spatial averages of c_v and c_r, the effective vertical and radial coefficients of consolidation, $c_{e,v}$ and $c_{e,r}$ are used to calculate the average degree of consolidation (U_{avg}) in spatially variable soil. To decouple the complex interaction between the governing RF and the numerical consolidation analysis, the probability density functions (PDF) of $c_{e,v}$ and $c_{e,r}$ are derived from the RF and used as random variables in the probabilistic consolidation analyses. Although the two representative values are random variables, the spatial variability can be reflected as their statistical properties are estimated through random fields. Two approaches to substitute MCS for the numerical analyses were investigated to maximize computational efficiency. The first approach uses the response surface function (RSF) in a limit state equation within the first-order reliability method (FORM) to estimate the probability of not meeting the target degree of consolidation, U_{tar}, termed the probability of under-consolidation, p_u. The second approach performs the SRSM to obtain the PDF of U_{avg} to determine p_u. The proposed approaches were verified by comparison to results obtained using the computationally-expensive MCS approach for two examples, and the impact of spatial variability of c on consolidation was evaluated. This works shows that it is possible to perform computationally efficient probabilistic analysis using two proposed approaches, and the spatial variability of c could be effectively considered using the representative c.

2. Theoretical background

2.1. Development and use of random fields

The spatial correlation of soil properties is known to influence the geotechnical response of soil and soil-supported structures (e.g., Fenton and Griffiths, 2008; Stuedlein et al., 2012b; Stuedlein and Bong, 2017). These uncertain spatial properties may be modeled by RFs rather than random variables. Vanmarcke and Grigoriu (1983) proposed the scale of fluctuation (SOF or δ), to describe the spatial extent over which a soil property shows strong spatial correlation. Autocorrelation has been used to express spatial changes in field-measured soil properties and the degree of dependency among neighboring observations. DeGroot and Baecher (1993) used the autocovariance distance, defined as the distance to which the autocovariance function decays to $1/e$ (where e is the base of the natural logarithm) to describe the extent of strong correlation. Numerous other autocorrelation functions have been proposed for use in geotechnical problems (e.g., Rackwitz, 2000). The single exponential autocorrelation function commonly used in geotechnical engineering analyses is used in this study and is given by:

$$\rho(x_1, x_2) = \exp\left(-\frac{|x_1 - x_2|}{l}\right) \tag{1}$$

where l represents the autocorrelation distance, and different autocorrelation distances in the vertical and horizontal directions is given by:

$$\rho(x, y) = \exp\left(-\frac{|x_1 - x_2|}{l_h} - \frac{|y_1 - y_2|}{l_v}\right) \tag{2}$$

where l_h and l_v represents the autocorrelation distances in the horizontal and vertical directions, respectively. The SOF implied by the single exponential autocorrelation function is equal to twice the value of the autocorrelation distance ($\delta = 2l$). In following analyses, the SOF was used to express the autocorrelation distance component of spatial variability.

The discretization of the RF is necessary for use in numerical analyses that use discrete grid or mesh distributions, and a methodology is necessary to represent the continuous RFs in terms of a vector of random variables. Several techniques have been developed to produce discrete RFs, including the midpoint method (e.g., Der Kiureghian and Ke, 1988), the spatial averaging method (e.g., Matthies et al., 1997), and the shape function method (e.g., Liu et al., 1986). However, these methods are relatively inefficient, and a large number of random variables (i.e., small grid sizes) are required to achieve a good approximation of the continuous RF. Series expansion methods have been developed to address the computation demands, and produce the most efficient RF discretization for a desired level of accuracy. The KLE is preferred for RF discretization when an exponential autocorrelation function is used because it provides the greatest accuracy (Sudret and Der Kiureghian, 2000).

The KLE uses a spectral representation of an RF, and the expansion is based on the spectral decomposition of its autocovariance function. The KLE of a RF may be represented using the mean value (μ_ω) and variance ($\sigma_\omega{}^2$) of the property and is given by (Spanos and Ghanem, 1989):