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A new depth-averaged model for flow-like landslides over complex terrains with curvatures and steep slopes

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ABSTRACT

Flow-like landslides are one of the most catastrophic types of natural hazards due to their high velocity and long travel distance. They travel like fluid after initiation and mainly fall into the 'flow' movement type in the updated Varnes classification (Hungr et al., 2014). In recent years, depth-averaged models have been widely reported to predict the velocity and run-out distance of flow-like landslides. However, most of the existing depth-averaged models present different shortcomings for application to real-world simulations. This paper presents a novel depth-averaged model featured with a set of new governing equations derived in a mathematically rigorous way based on the shallow flow assumption and Mohr-Coulomb rheology. Particularly, the new mathematical formulation takes into account the effects of vertical acceleration and curvature effects caused by complex terrain topographies. The model is derived on a global Cartesian coordinate system so that it is easy to apply in real-world applications. A Godunov-type finite volume method is implemented to numerically solve these new governing equations, together with a novel scheme proposed to discretise the friction source terms. The hydrostatic reconstruction approach is implemented and improved in the context of the new governing equations, providing well-balanced and non-negative numerical solutions for mass flows over complex domain topographies. The model is validated against several test cases, including a field-scale flow-like landslide. Satisfactory results are obtained, demonstrating the model's improved simulation capability and potential for wider applications.

1. Introduction

Flow-like landslides are one of the most catastrophic types of natural hazards due to their high velocity and long travel distance. They travel like fluid after initiation and mainly fall into the 'flow' movement type in the updated Varnes classification (Hungr et al., 2014). Numerical models have been widely used to predict the dynamics of flow-like landslides and hence quantify the run-out distance and flow velocity to facilitate risk assessment and management. Due to their much simplified formulation and less computational demand compared with the fully 3D models, depth-averaged models have been widely reported and successfully applied to simulate granular flows, including flow-like landslides. Savage and Hutter (1989) made the first attempt to develop a depth-averaged model for granular flows based on the Mohr-Coulomb internal rheology law and constant Coulomb bed friction. Their approach has since been adopted and extended by numerous researchers to develop granular flow models (e.g. Hungr, 1995; Iverson, 1997; Gray et al., 1999; Iverson and Denlinger, 2001; Denlinger and Iverson, 2001; Gray et al., 2003; Mcdougall and Hungr, 2004; Denlinger and Iverson, 2004; Pudasaini and Hutter, 2003; Pudasaini et al., 2005; Mangeney et al., 2007; Luca et al., 2009, 2012; Gray and Edwards, 2014; Edwards and Gray, 2014; Iverson and George, 2014; George and Iverson, 2014).

For geophysical granular flows such as avalanches, landslides and debris flows, a challenging task is to simulate the real-world events taking into account the effect of complex 3D topographies. A major difference between the flow-like landslides and water flows, such as river flows or overland flood waves, is that flow-like landslides usually take place on steep slopes rather than nearly horizontal and flat ground surface. This poses a major challenge in developing depth-averaged models. As shown in Fig. 1, over a steep slope, the vertical acceleration ($\mathbf{a}_v$) of a particle or mass element is non-zero. Consequently, the pressure distribution along the vertical direction can no longer be trivially calculated in the same way as the conventional shallow water equations defined on the Cartesian coordinates. In addition, there exists a centrifugal force ($\mathbf{a}_c$) along the direction normal to the bed when it is curved. Incorporating the vertical acceleration and centrifugal force into the depth-averaged models is essential for accurate simulation of granular flows over complex terrains. This has been a challenge since

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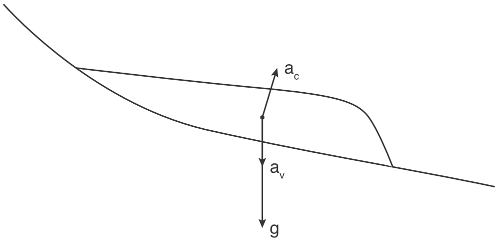

Fig. 1. Vertical acceleration and centrifugal force for a flow-like landslide running on a steep slope.

the very first depth-averaged model was developed.

When deriving a depth-averaged model, the flow direction is normally assumed to be parallel to the domain surface, so that the normal pressure term is trivial to calculate. As a result, surface-fitted curvilinear coordinate systems have been widely adopted for developing depth-averaged granular flow models. The first of such attempts was made by Gray et al. (1999) through the introduction of an orthogonal curvilinear coordinate system to develop their model; but their model assumes that predominant topographic variation only occurs in one direction and so faces a major difficulty in adapting to real-world terrains with more complex topographic features. Later on, Pudasaini and Hutter (2003) introduced a model on a non-orthogonal curvilinear coordinate system for avalanches in arbitrary curved and twisted channels. Such a non-orthogonal curvilinear coordinate system has also been adopted in other granular flow models (e.g. Pudasaini et al., 2005, 2007) and achieved certain level of success.

A curvilinear coordinate system relies on the thalweg along the bed to define its main axis. For a real-world complex topography, however, it is usually difficult to define the downslope direction or the thalweg due to large variations of the topography in different directions. Furthermore, to facilitate the simulation of geophysical flows (e.g. landslides, debris flows and avalanches) in the real world, Digital Elevation Models (DEMs) are commonly used to describe the terrain topography, which is commonly defined on a Cartesian coordinate system. Transformation of topographic data from the Cartesian coordinate system to the curvilinear coordinate system must be applied, inevitably increasing computational effort and leading to a loss of accuracy, particularly in the cases where the topographies are featured with abrupt changes. To avoid this, Bouchut and Westdickenberg (2004) introduced a shallow water flow model on an arbitrary coordinate system for simulations over topographies with small curvatures. Their overall governing equations were derived on a fixed Cartesian coordinate system while the variables were defined on a local reference coordinate system aligning with the local topography, i.e. the flow depth is normal to and the velocities are parallel to the bed. This model was later extended and applied to granular flows by Mangeney et al. (2007). In their model, the terrain topographies may be directly described by a DEM, but coordinate transformation is still needed to provide the initial depth along the vertical axis. For real-world applications with complex topographies, defining flow depth normal to bed can be inconvenient because performing such coordinate transformation is not only time-consuming but also sometimes difficult, if not impossible, especially when discontinuous topographies arise.

In practice, it is desired to develop a model based on a global Cartesian coordinate system in which the vertical axis is aligned with the gravity direction so that DEMs can be directly used to support model setup without the need of coordinate transformation. But a global Cartesian coordinate system based model may also have its limitations, as the determination of pressure/stress terms becomes a much more difficult task within such a configuration. Granular flows commonly happen on steep inclined slopes. The flow acceleration along the vertical direction has a magnitude comparable to the gravity and is not negligible (Iverson, 2014). The vertical normal pressure thus becomes more difficult to calculate. The centrifugal force caused by bed surface curvature is also no longer trivial to quantify because the velocity variables are not defined parallel to the terrain surface.

In order to utilise a global Cartesian coordinate system and meanwhile maintain solution accuracy, Denlinger and Iverson (2004) subtracted the vertical acceleration from the gravity acceleration to account for the non-hydrostatic pressure effect; their model provided better results than a hydrostatic model when applying to a granular dam break test. More recently, Castro-Orgaz et al. (2014) suggested that the Boussinesq-type models which retain the non-hydrostatic pressure to a certain level and have been successfully applied in modelling shallow water waves may be also implemented for simulating gravity-driven granular flows. They also pointed out that Denlinger and Iverson's model can actually be categorized as a Boussinesq-type model for granular flow. The model by Castro-Orgaz et al. (2014) has recently been further simplified by Yuan et al. (2017) and implemented in an existing code. These Boussinesq-type non-hydrostatic models generally predict better results than their hydrostatic counterparts. However, more sophisticated numerical schemes must be used to solve the Boussinesq-type governing equations due to the presence of additional higher-order derivative terms. Therefore, a Boussinesq-type model is computationally much more demanding and usually less stable than a model solving the shallow water equations or similar depth-averaged formulations.

Other simpler global Cartesian coordinate system based models have also been reported (e.g. Juez et al., 2013; Hergarten and Robl, 2015). These models simply modified the original shallow water type equations by including a projection factor to the pressure and source terms, determined by the consideration of bed or surface topography gradients according to heuristic geometric arguments. These models produce very similar results to those models based on local curvilinear coordinate systems, which has been confirmed by the authors' previous study (Xia et al., 2015). Compared with the Bousinessq-type models, the numerical implementation of these shallow water type models can be much easier to achieve and many well-documented numerical schemes developed for shallow flow hydrodynamics can be directly used. However, although these models have proven to be successful for certain applications, they have not been fully justified in a mathematically rigorous way and all of them do not consider the effect of the centrifugal force induced by bed curvatures which may become significant for applications involving complex topographies.

Numerous robust numerical schemes have been reported in the literature for solving the shallow water equations in the context of hydrodynamic simulation. In the last two decades, particular attention has been paid to develop shock-capturing numerical schemes to support accurate and stable simulation of shallow flow hydrodynamics over dry terrains with complex topographies (e.g. Gray et al., 2003; Audusse et al., 2004; Liang and Marche, 2009; Hou et al., 2014). Such numerical schemes are generally required to maintain the *C*-property (i.e. preserving the lake at rest solution at the discrete level) and include proper numerical techniques to handle wet/dry interface and discretise the friction source terms. However, some of these issues (e.g. *C*-property and friction term discretisation) have not been thoroughly considered and resolved in the context of flow-like landslide modelling, which calls for more research efforts (e.g. Mangeney et al., 2007; Juez et al., 2013; Zhai et al., 2015).

In order to correctly take into account the effects of large slope gradients and curvatures, but meanwhile allow the users to directly take advantages of DEM data, this work presents a new depth-averaged model based on a global Cartesian coordinate system with the following highlights:

1. New depth-averaged equations are derived through depth-integration and asymptotic analysis, taking into account the effects of vertical acceleration and centrifugal force; the resulting equations are hyperbolic and rotationally invariant, and mathematically