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Technical note

The hydraulic conductivity of sediments: A pore size perspective

X.W. Ren[a,*], J.C. Santamarina[b]

[a] Faculty of Engineering, China University of Geosciences, Wuhan 430074, China
[b] Earth Science and Engineering, KAUST, Saudi Arabia

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ABSTRACT

This article presents an analysis of previously published hydraulic conductivity data for a wide range of sediments. All soils exhibit a prevalent power trend between the hydraulic conductivity and void ratio. Data trends span 12 orders of magnitude in hydraulic conductivity and collapse onto a single narrow trend when the hydraulic conductivity data are plotted versus the mean pore size, estimated using void ratio and specific surface area measurements. The sensitivity of hydraulic conductivity to changes in the void ratio is higher than the theoretical value due to two concurrent phenomena: 1) percolating large pores are responsible for most of the flow, and 2) the larger pores close first during compaction. The prediction of hydraulic conductivity based on macroscale index parameters in this and similar previous studies has reached an asymptote in the range of $k_{meas}/5 \le k_{predict} \le 5k_{meas}$. The remaining uncertainty underscores the important role of underlying sediment characteristics such as pore size distribution, shape, and connectivity that are not measured with index properties. Furthermore, the anisotropy in hydraulic conductivity cannot be recovered from scalar parameters such as index properties. Overall, results highlight the robustness of the physics inspired data scrutiny based Hagen–Poiseuille and Kozeny-Carman analyses.

1. Introduction

Theoretical and empirical equations relate the hydraulic conductivity of soils to properties such as the grain size, specific surface area, clay content, porosity and pore geometry (Taylor, 1948; Malusis et al., 2003; Zhang et al., 2005; Roque and Didier, 2006; Dolinar, 2009; Mejias et al., 2009; Chapuis, 2012; Wang et al., 2013; Ilek and Kucza, 2014; Sante et al., 2015; Ren et al., 2016; Kucza and Ilek, 2016). The analytically derived Kozeny-Carman (KC) equation considers the porous network in sediments as a bundle of tubes and assumes Poiseuille's laminar fluid flow in the tubes. The sediment hydraulic conductivity k [m/s] can then be expressed in terms of the sediment specific surface area S_s [m^2/g] and void ratio e (Taylor, 1948):

$$k = \frac{C_F\, g}{\nu_f \cdot \rho_m^2} S_s^{-2} \frac{e^3}{1+e} \tag{1}$$

where ρ_m [kg/m^3] is mineral mass density, ν_f [m^2/s] is the kinematic fluid viscosity, and $C_F \approx 0.2$ is a constant related to pore topology. In general, it is thought that the Kozeny-Carman equation more accurately predicts trends in hydraulic conductivity for coarse-grained sandy sediments than for fine-grained clayey soils.

Empirical correlations have been suggested for coarse sandy sediments and for fine-grained clayey sediments. Hazen's equation is the most frequently cited empirical equation for coarse-grained soils and emphasizes the role of the finer fraction on a soil's hydraulic conductivity (Hazen, 1892):

$$\frac{k}{cm/s} \approx \left(\frac{D_{10}}{mm}\right)^2 \tag{2}$$

where the grain size D_{10} corresponds to the finer 10% of the soil mass (Note: the temperature correction in the original equation is not included here because prediction errors overwhelm the temperature effects). Predicted and measured values can differ in more than one order of magnitude because of grain size variability and particle shape (Lambe and Whitman, 1969; Shepherd, 1989; Carrier, 2003). Other size fractions have been considered to enhance predictability, such as D_5, D_{20} or D_{50}, however the original function of D_{10} remains best known (Sherard et al., 1984; Kenney et al., 1984; Indraratna et al., 2012). Hazen's first-order estimate of hydraulic conductivity was based on poorly graded sands packed at medium density, and is independent of the void ratio e in part due to the low compressibility of coarse grained sediments (Note: Taylor, 1948 corrected the computed values for void ratio, following the Kozeny-Carman's equation).

Empirical equations for fine-grained soils explicitly recognize the dependency of hydraulic conductivity on the void ratio. Two forms have been proposed: (a) an exponential or log-linear relation (Taylor,

* Corresponding author.
E-mail addresses: xingweiren@cug.edu.cn (X.W. Ren), carlos.santamarina@kaust.edu.sa (J.C. Santamarina).

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1948; Nishida and Nakagawa, 1969; Lambe and Whitman, 1969; Mesri and Rokhsar, 1974; Tavenas et al., 1983a, 1983b),

$$\log[k/(cm/s)] = a + b\cdot e \tag{3}$$

and (b) a power relation (Mesri and Olson, 1971; Samarasinghe et al., 1982; Carrier and Beckman, 1984; Krizek and Somogyi, 1984; Dolinar, 2009)

$$k = \alpha\, e^{\beta} \tag{4}$$

Model parameters in both cases have been related to either the liquid limit or the plastic limit of the soil (e.g., Carrier and Beckman, 1984; Berilgen et al., 2006; Dolinar and Škrabl, 2013); Furthermore, given the parallelism between Eq. (3) and Terzaghi's compressibility equation $e = e_o - C_c\log(\sigma'/\sigma_o)$, model parameters a & b can also be associated with the sediment compressibility (Mesri and Rokhsar, 1974; Tavenas et al., 1983a, 1983b; Nagaraj et al., 1994). Exponential and power equations have been used for geomaterials that range from suspensions (Pane and Schiffman, 1997), to normal and over-consolidated clays (Al-Tabbaa and Wood, 1987; Nagaraj et al., 1994), and rocks (David et al., 1994).

This study reexamines the hydraulic conductivity of sediments. The study includes an extensive compilation of published data gathered for a wide range of sediments. The subsequent analysis seeks to identify the causal link between physics-based theoretical models and the observed empirical trends, and to identify the underlying pore-scale processes that can justify prevailing trends and anticipate potential limitations and deviations.

2. Data compilation - the central role of pore size

The hydraulic conductivity database includes both natural and remolded sediments (coarse gravels to smectite clays, and mixtures), and of different fabrics (loose and dense packing and both flocculated and dispersed). The database is plotted in all linear-log scale combinations. We note that individual datasets plot as linear trends on the log(k)-log(e) space as shown in Fig. 1. These data show two opposite trends for hydraulic conductivity as a function of void ratio: while the hydraulic conductivity increases with increasing void ratio for any single sediment, fine-grained (small pore size) soils exhibit much lower hydraulic conductivity -even at higher void ratios- than coarse-grained (large pore size) sediments. In fact, the Kozeny-Carman equation highlights the importance of "pore size" rather than "porosity" on fluid transport (anticipated by the Hagen–Poiseuille equation for a single tube).

2.1. The relevance of macroscale values e and S_s

The apparent contradiction in the last statement points to the importance of "pore size" rather than "porosity" on fluid transport. The mean pore size d_p can be computed for various grain geometries and fabrics in terms of the void ratio e and the specific surface area S_s [m^2/g] (Phadnis and Santamarina, 2011). Consider the volume of voids evenly distributed around grains as a "void layer" of thickness t_{void}; the inter-particle distance $d_p = 2t_{void}$ is then considered as an estimate of the mean pore size

$$d_p = 2\frac{e}{S_s\rho_m} \tag{5}$$

where ρ_m [kg/m^3] is the mineral mass density. This first order estimate of pore size is based on two macroscale parameters: void ratio e and specific surface area S_s.

The specific surface area is not reported in most studies plotted in Fig. 1. We estimate the specific surface area by using other published soil descriptions. In the case of fine grained soils, estimates were based on liquid limits w_L (Farrar and Coleman, 1967; see also Muhunthan, 1991 and Santamarina et al., 2002b),

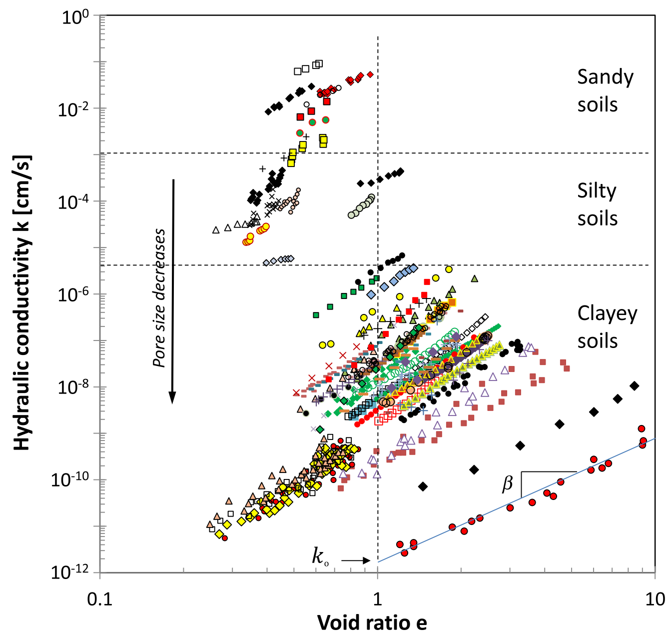

Fig. 1. Hydraulic conductivity versus void ratio. Data gathered for natural and remolded sediments, from coarse sands to fine-grained clays and various fabrics. Dataset: 1440 data points, 92 soils.
Data sources: Mesri and Olson, 1971; Horpibulsuk et al., 2011; Michaels and Lin, 1954; Raymond, 1966; Siddique and Safiullah, 1995; Tavenas et al., 1983a, 1983b; Terzaghi et al., 1996; Deng et al., 2011; Dolinar, 2009; Sanzeni et al., 2013; Kwon et al., 2011; Kim et al., 2013; Sridharan and Nagaraj, 2005a, 2005b; Bandini and Sathiskumar, 2009; Sivapullaiah et al., 2000; Chu et al., 1954; Pandian, 2004; Kaniraj and Gayathri, 2004; Taylor, 1948; Chapuis et al., 1989; Lambe and Whitman, 1969.

$$S_s = 1.8 w_L - 34 \tag{6}$$

For sandy soils made of rotund grains, the specific surface area was estimated from the cumulative grain-size distribution (Santamarina et al., 2002b - assumes linear distribution in log scale),

$$S_s = \frac{3}{4}\frac{C_u + 7}{\rho_w G_s D_{50}} \tag{7}$$

where the coefficient of uniformity is $C_u = D_{60}/D_{10}$, and D_{10}, D_{50} D_{60} [mm] are the grain diameters for 10%, 50% and 60% cumulative passing fractions. The specific surface area in mixtures is estimated as a summation of the surface area contributed by the various size fractions weighted by their mass fractions.

2.2. Hydraulic conductivity vs. pore size

We use Eq. (5) to estimate the mean pore size for the data set plotted in Fig. 1. Hydraulic conductivity data are then replotted as a function of the computed mean pore size d_p, in Fig. 2. Additional data for sands with known grain size distributions are included in this figure. We observe: (1) all experimental data gathered for soils ranging from coarse- to fine-grained sediments collapse onto a relatively narrow single trend in the k-d_p space; (2) the Hagen–Poiseuille equation for fluid flow in cylindrical tubes predicts a power-2 relationship between hydraulic conductivity and pore size $k \propto d_p^2$. The line with slope 2 superimposed on the data in Fig. 2 closely agrees with the overall trend.

These observations confirm the central role of pore size on hydraulic conductivity. Furthermore, the analysis presented above demonstrates the relevance of the two measurable macroscale parameters, the void ratio e and specific surface area S_s, as captured in the Kozeny-Carman Eq. (1).