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Computational homogenization for mechanical properties of sand cobble stratum based on fractal theory

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ABSTRACT

Sandy cobble strata is one of the most commonly encountered engineering geological body. A further research into the stability of ground and underground structure demands for precise modeling of actual formation. In this paper, a meso-scale numerical method for modelling sandy cobble strata is presented, which considers rock blocks and soil matrix as separate constituents. In the approach, the particle size distribution and spatial distribution of rock blocks are two crucial points. For the first point, i.e. the particle size distribution of rocks, the fractal scaling theory is included in particle size distribution description and design. In the log-log plots, 60 sets of particle size distributions of sandy cobble soil in Beijing are examined for fractal behavior. For the second point, i.e. the spatial distribution of rocks, the Monte-Carlo principle is used to generate random spatial distribution of rocks in soil matrix, while the rock shape is assumed to be circular in two dimension, or sphere in three dimension. Based on the compassion of rock volume content between the numerical model and theoretical calculation value, it is validated that the meso-scale method could basically be used to establish the meso model of sandy cobble strata. Then, the present sandy cobble strata model is extended to investigate tunnel excavation in such a formation. Based on the simulation results, the deformation characteristics on transverse section and longitudinal profile are explored respectively, and their variations with fractal dimension or the maximum rock size are discussed subsequently.

1. Introduction

Nowadays, demand for public transportation has grown considerably, which inevitably need to carry out construction in complicated geological conditions. Sandy cobble stratum, which is composed of heterogeneous soil-rock mixture (SRM) (Lindquist, 1994; Medley, 1994; Wakabayashi and Medley, 2004), is one of the most commonly encountered complex engineering geological body. When constructing in this formation, the performance of ground and underground structure are different from that in homogeneous layers. A better understanding on them needs to develop an appropriate analysis method and a specific modelling theory of actual formation.

Generally, the sandy cobble strata is considered as a homogeneous medium. In the efforts of Chen and Zhang (2005), Migliazza et al. (2009), Luo et al. (2013), Zhang et al. (2013), Fargnoli et al. (2015), Katebi et al. (2015), Małkowski (2015), Xiao et al. (2016), Zhang et al. (2016), macroscopic homogeneous numerical models of sandy cobble stratum were employed to investigate tunnel excavation or slope stability. In their simulations, the material is represented in a homogenized way using effective material properties. However, a closer look reveals that the particle in sandy cobble strata ranging from millimeter-sized grains to meter-sized boulders. Sandy cobble soil is not a homogenous material in nature, it has a heterogeneous internal structure. On the other hand, limited by experimental apparatus, the test specimen can hardly cover all the particles, which yields significantly error for effective material properties. So, though homogeneous model achieves a high efficiency, it has limits in reflecting internal characteristics of actual formation.

Treating sandy cobble soil as a series of particles, some researchers (He et al., 2012, 2013; Shi et al., 2013a,b) developed sandy cobble formation model using discrete element method. It is known that the determination of particles micro-parameters should combine with experiment results. Thus, similar to homogeneous model, this method has same troubles in obtaining reasonable test data for parameter calibration. Meanwhile, because the contact detection algorithm between particles is very complex and time consuming, the application of discrete element model is always limited to small specimens.

On the meso-scale, sandy cobble strata can be treated as a composite material formation composed of fine-grained soils and large-sized hard rocks, with or without jointing between them. For each individual

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component, i.e. soil matrix and rock, there are considerable difference in their mechanical properties, such as elastic modulus, shear strength, friction angle and cohesion. Considering the heterogeneity character of material, two main concepts for establishing a numerical meso-scale model of sandy cobble strata can be distinguished, in which the shape and the spatial distribution of the rocks are represented explicitly. The first way is based on image processing techniques (Xu et al., 2008; Shi et al., 2013a,b; Wang et al., 2015). The second approach, followed in this work, is the artificial generation of the microstructure. Based on the statistical data of SRM meso-structure, Li et al. (2014) established stochastic models coupling with Autocad. Treating rock as tetrahedron or hexahedron, Xu et al. (2015) developed a three dimensional (3D) random meso-structure generation system of SRM. Incorporating the algorithm of Entrance block A and block B, an enhanced random sequential addition procedure was built to generate SRM random aggregate structure (Chen et al., 2017). Great process has been made in meso-modeling theory and analysis of concrete (Unger and Eckardt, 2011; Du et al., 2013; Yang and Wang, 2015), porous material (Zhuang and Zhu, 2015), pre-cracked brittle material (Zhu et al., 2016) and nanocomposites (Vu-Bac et al., 2015), which provides significant reference for sandy cobble strata modeling.

The aim of the present study is to explore a meso-scale approach for establishing sandy cobble stratum model and illustrate its application in tunneling. Taking the material heterogeneity into account, the sandy cobble soil in the present work was treated as a two-phase composite. The meso-scale material constituents involved the soil matrix and rock blocks. The soil matrix was considered homogeneous, while the rocks were regarded as spheres in 3D or circulars in 2D. Fractal theory was employed to describe the particle size distribution of rocks in soil matrix, while the spatial distribution of these rocks followed a classical algorithm called Monte-Carlo principle.

The article is outlined as follows. In the next section, the fractal effect in the particle size distribution of soil is described in detail, including the fractal characteristics analysis of sandy cobble soil in Beijing. On the basis of this, Section 3 presents an algorithm for the generation of sandy cobble strata model. Application of the established meso-model in tunneling is presented in Section 4. Finally, Section 5 shows some conclusions.

2. Fractal effect in the particle size distribution of sandy cobble soil

To achieve a rational numerical model, the actual size distribution of particles had better be presented in the meso-model. Usually, the soil particle grading curve is used to describe particle size distribution. However, due to the limit of engineering exploration technology, it is difficult to get a complete grading curve from sieve analysis test.

Fortunately, studies (Avnir et al., 1985; Turcotte, 1986; Wu et al., 2014) showed that the self-similar characteristic i.e. the invariance of fractal dimension regardless of the scale of observation, had been observed in soil particle size distribution for natural soils. This feature is in accordance with the fractal theory, providing a theoretical basis for grading description and design. When the particle size distribution confirms to the fractal distribution law, respecting it could reduce modeling workloads and improve the approximation of simulation result to the actual state.

2.1. Description of particle size distribution based on the fractal theory

The grading of the particles can be explicitly given based on the fractal theory. For a soil particle size distribution, the fractal theory suggests that across a wide range of scales, solid phase will appear self-similarity. For two-dimensional case, the fractal behavior can be illustrated as follows.

The area A needed to cover the grains of size r larger than a specific measuring scale, R, is given by Mandelbrot (1991) as follows:

$$A(r > R) = C_a\left[1 - \left(\frac{R}{\lambda_a}\right)^{2-D}\right] \tag{1}$$

where C_a is the area shape factors, λ_a is the largest grain size and D is the fractal dimension for the two dimensional case.

Carrying the analogy to three dimensions, the volume $V(r > R)$ that needed to fill the soil grains of size R or larger is given by Tyler and Wheatcraft (1992):

$$V(r > R) = C_v\left[1 - \left(\frac{R}{\lambda_v}\right)^{3-D}\right] \tag{2}$$

where C_v is the volume shape factors, λ_v is the largest grain size and D is the granularity fractal dimension for the three dimensional case.

As the variation in particle density as a function of grain size is seldom reported, the particle density is therefore taken to be constant. Letting both sides multiplying by ρ_P, one can obtain that the mass M $(r > R)$ is

$$M(r > R) = \rho_p C_m\left[1 - \left(\frac{R}{\lambda_m}\right)^{3-D}\right] \tag{3}$$

The total mass, M_T, thus can be given by

$$M_T = M(r > 0) = \rho_p C_m\left[1 - \left(\frac{R}{\lambda_m}\right)^{3-D}\right] \tag{4a}$$

$$M_T = \rho_p C_m \tag{4b}$$

Eq. (3) can be normalized by Eq. (4b) to yield

$$\frac{M(r > R)}{M_T} = 1 - \left(\frac{R}{\lambda_m}\right)^{3-D} \tag{5}$$

The constant λ_m can be evaluated if the largest grain size R_L is selected for fractal behavior. At $R = R_L$, $M(r > R)/M_T = 0$ and λ_m must be equivalent to R_L. Thus, Eq. (5) can be converted into

$$\frac{M(r < R)}{M_T} = \left(\frac{R}{R_L}\right)^{3-D} \tag{6}$$

Eq. (6) is the typically reported particle size distribution date of "percentage of mass less than".

When drawn in a log-log plot, the function can be written in the following form that

$$\log\left(\frac{M(r < R)}{M_T}\right) = (3 - D)\log\left(\frac{R}{R_L}\right) \tag{7}$$

Drawing the particle size distribution in the log-log plot, if the fitting line shows a good linear correlation, one can obtain that the soil is examined for fractal behavior. The slope of the fitting straight line is subsequently identified to be k, and then the fractal dimension of soil $D = 3 - k$ could be obtained.

It can be seen from above, fractal dimension and the maximum rock size are two key parameters in describing soil particle size distribution. Therefore, when given these two parameters, the particle size distribution curve will be presented as a mathematical formula.

2.2. Statistic fractal characteristic of sandy cobble soil in Beijing

In the western Beijing, almost all the subway lines are conducted in sandy cobble stratum. This formation contains more than 60% pebbles (particles larger than 20 mm) and lenses of fines or sand. During the construction of subway tunnels, lots of sandy cobble soil samples were collected in detail geological surveys. Among that, 60 groups of particle size distributions on mass (Beijing Urban Construction Exploration and Surveying Design Research Institute Co. Ltd., 2007a,b; Guo, 2007; Zhang and Chen, 2007; Wang, 2011; Pan and Lei, 2012; Zhou, 2012; Jiang et al., 2013; Li, 2013; Su et al., 2013; Li et al., 2015) were selected