Science Bulletin 63 (2018) 765–770

Contents lists available at ScienceDirect

Science Bulletin

journal homepage: www.elsevier.com/locate/scib

Article

Operational effects of the UNOT gate on classical and quantum correlations

Kuan Zhang [a,*], Jiajun Ma [a,b], Xiang Zhang [c,a], Jayne Thompson [d], Vlatko Vedral [b,d,e,a], Kihwan Kim [a,*], Mile Gu [f,g,d,a,*]

[a] Center for Quantum Information, Institute for Interdisciplinary Information Sciences, Tsinghua University, Beijing 100084, China
[b] Department of Atomic and Laser Physics, Clarendon Laboratory, University of Oxford, Oxford OX1 3PU, UK
[c] Department of Physics, Renmin University of China, Beijing 100872, China
[d] Centre for Quantum Technologies, National University of Singapore, Singapore 117543, Singapore
[e] Department of Physics, National University of Singapore, Singapore 117551, Singapore
[f] School of Mathematical and Physical Sciences, Nanyang Technological University, Singapore 637371, Singapore
[g] Complexity Institute, Nanyang Technological University, Singapore 637335, Singapore

ARTICLE INFO

Article history:
Received 29 March 2018
Received in revised form 3 May 2018
Accepted 8 May 2018
Available online 18 May 2018

Keywords:
Quantum information
Universal NOT gate
Quantum correlations
Quantum discord
Data processing inequality
Embedding quantum simulation
Ion trap

ABSTRACT

The NOT gate that flips a classical bit is ubiquitous in classical information processing. However its quantum analogue, the universal NOT (UNOT) gate that flips a quantum spin in any alignment into its antipodal counterpart is strictly forbidden. Here we explore the connection between this discrepancy and how UNOT gates affect classical and quantum correlations. We show that while a UNOT gate always preserves classical correlations between two spins, it can non-locally increase or decrease their shared discord in ways that allow violation of the data processing inequality. We experimentally illustrate this using a multi-level trapped $^{171}Yb^{+}$ ion that allows simulation of anti-unitary operations.

1. Introduction

When given a quantum spin pointing in some unknown direction $\vec{n}$, is it possible to engineer a universal device that flips this spin to point in the antipodal direction $-\vec{n}$? While this process is easy to envision for classical vectors, it is strictly impossible for quantum spins. The quantum operation that takes an arbitrary quantum state $|\varphi\rangle$ to its orthogonal complement $|\varphi^{\perp}\rangle$ is anti-unitary, and thus does not exist [1–4]. Like the no-cloning theorem, this uniquely quantum constraint has drawn significant scientific interest [5–7].

In contrast to cloning, the radical operational consequences of the UNOT gate are not as readily apparent on a single qubit. Suppose Alice secretly encodes a direction $\vec{n}$ in 3-dimensional space by preparing a spin aligned in $\vec{n}$. She then challenges Bob to estimate $\vec{n}$. If Bob can perfectly clone quantum states, then he can violate the uncertainty principle by measuring each clone in a different complementary basis. On the other hand, any measurement Bob makes after applying a UNOT gate on the input spin can be simulated by measuring the input directly and reinterpreting the measurement outcome (recording "up" as "down" and vice versa). Thus UNOT gates do not allow Bob to retrieve information about $\vec{n}$ beyond standard quantum limits.

The consequences of the UNOT gate surface when an ancillary qubit is introduced. Consider the same game, but now played on two qubits. Instead of sending a single spin, Alice now sends a pair of spins. Take two different strategies, either (1) sending Bob two aligned spins, both in direction $\vec{n}$, or (2) an anti-aligned pair, with one spin in direction $\vec{n}$, and the other in direction $-\vec{n}$. Gisin and Popescu [1] illustrated that the second strategy improves Bob's capacity to estimate $\vec{n}$. They noted that if Bob possesses a UNOT gate, he can deterministically convert a pair of aligned spins to anti-aligned spins, and thus break standard quantum limits whenever Alice adopts strategy (1).

This connection suggests that UNOT gate exhibits distinctive effects on quantum correlations. Here we formalize this intuition using recent methods that isolate the purely quantum component of correlations between two systems. These correlations, known as discord, are often motivated as correlations accessible only to quantum observers [8–10]. We show that the UNOT gate preserves

* Corresponding authors.
E-mail addresses: zhangkuan13@gmail.com (K. Zhang), kimkihwan@mail.tsinghua.edu.cn (K. Kim), cqtmileg@nus.edu.sg (M. Gu).

https://doi.org/10.1016/j.scib.2018.05.011
2095-9273/

classical correlations between two spins, but can change their shared quantum correlations in ways forbidden by fundamental data processing principles (Fig. 1a). We illustrate this through experiment — by adapting recent ion trap technology that allows perfect simulation of anti-unitary operations [11–13]. We then outline how these results rationalize the discrepancy in communication rate between aligned and anti-aligned spins, showing that it exactly relates to the UNOT gate's non-trivial effect on discord during the decoding process.

2. Theory

2.1. Technical framework

Consider first two classical spins A and B. Let $S(\cdot)$ denote the information entropy function, such that $S(\mathrm{A})$ and $S(\mathrm{B})$ quantify the respective uncertainties of A and B when viewed independently and $S(\mathrm{AB})$ the uncertainty of the joint spin pair. The mutual information $I(\mathrm{A},\mathrm{B}) = S(\mathrm{A}) + S(\mathrm{B}) - S(\mathrm{AB})$ then captures the total correlations between A and B. This coincides with

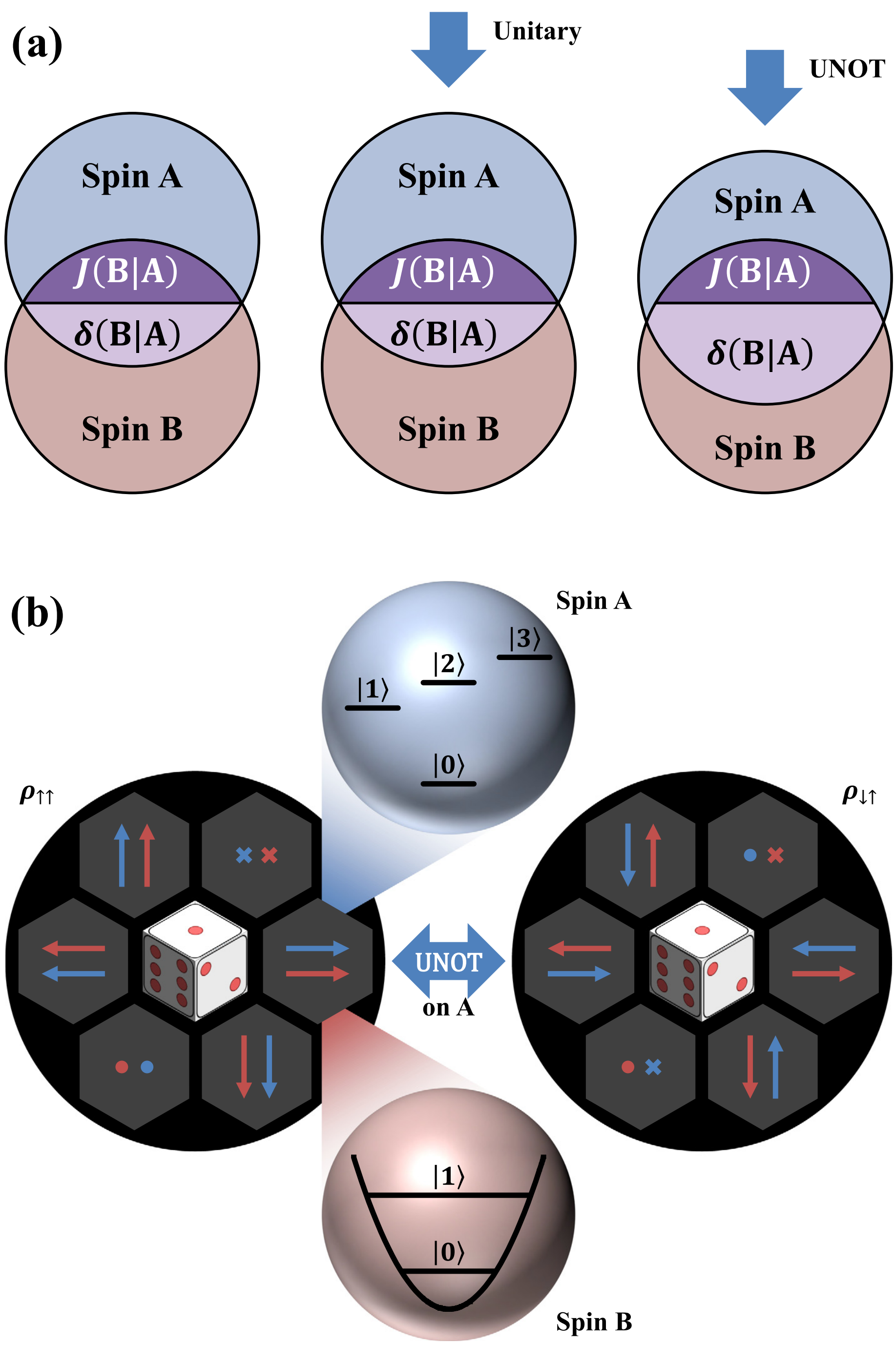

Fig. 1. UNOT gates on spin pairs. (a) Local reversible operations leave classical correlations $J(\mathrm{B}|\mathrm{A})$ and quantum correlations $\delta(\mathrm{B}|\mathrm{A})$ unchanged, where A and B are two separable spins. A UNOT gate also preserves $J(\mathrm{B}|\mathrm{A})$, but can change $\delta(\mathrm{B}|\mathrm{A})$. (b) We simulate the effect of UNOT on $\rho_{\uparrow\uparrow}$ and $\rho_{\downarrow\uparrow}$, where the two spins are encoded in the internal and external degrees of freedom of a trapped $^{171}\mathrm{Yb}^{+}$ ion in a harmonic potential. Spin A is mapped to a 4-level system using Eq. (1), spanned by the basis $|0\rangle_{\mathrm{A}} = |F=0, m_{\mathrm{F}}=0\rangle$ and $|n=1,2,3\rangle = |F=1, m_{\mathrm{F}}=n-2\rangle$, where F and m_{F} characterize the total internal angular momentum of $^{171}\mathrm{Yb}^{+}$. The transition frequency from $|F=0, m_{\mathrm{F}}=0\rangle$ to $|F=1, m_{\mathrm{F}}\rangle$ is $(2\pi)(12642.8 + 9.0m_{\mathrm{F}})$ MHz. Spin B is mapped to the ground and first excited states of external motional mode, denoted by $|0\rangle_{\mathrm{B}}$ and $|1\rangle_{\mathrm{B}}$, which are separated by the trap frequency $(2\pi)2.44$ MHz.