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Article

Emergent symmetry at superradiance transition of a Bose condensate in two crossed beam cavities

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ABSTRACT

Recently an experiment on superradiant transition of a Bose condensate in two crossed beam cavities has been reported by Léonard et al. in Nature 543, 87 (2017). The surprise is they find that across the superradiant transition, the cavity light can be emitted in any superposition of these two cavity modes. This indicates an additional $U(1)$ symmetry that does not exist in the full Hamiltonian. In this paper we show that this symmetry is an emergent symmetry in the vicinity of the phase transition. We identify all the necessary conditions that are required for this emergent $U(1)$ symmetry and show that this experiment is a special case that satisfies these conditions. We further show that the superradiant transition in this system can also be driven to a first order one when the system is tuned away from the point having the emergent symmetry.

1. Introduction

Symmetry plays a fundamentally important role in physics. The term of emergent symmetry refers to situations where the symmetry group for the low-energy physics of a quantum system can actually be larger than that of the full Hamiltonian. This usually requires fine tuning of certain parameters in the Hamiltonian. For instance, there are at least two known examples of emergent symmetry in ultracold atomic systems. The first is the emergent Lorentz symmetry in the Bose-Hubbard model, when the system is fine tuned to the particle-hole symmetric point and is in the vicinity of the superfluid to Mott-insulator transition [1–3]. One physical consequence of this emergent Lorentz symmetry is the appearance of the Higgs mode, which has been observed by Endres et al. [4]. Another example is the emergent non-relativistic conformal symmetry in a Fermi gas with a short-ranged interaction, when the interaction potential is fine tuned to a two-body resonance. This emergent conformal symmetry also has important physical consequences, such as the vanishing of the bulk viscosity [5–9].

Placing ultracold atoms into a cavity represents a new hybridized quantum system at the interface of quantum optics and many-body physics [10–12], and the inevitable decay of the cavity light makes the system intrinsically non-equilibrium. Previous experiments have studied Bose condensates in a single cavity, and it has been found that the pumping field can drive a superradiant transition, across which the cavity field becomes finite and the Bose condensate acquires a density wave order simultaneously [13–16]. Along this line of study, experimental and theoretical efforts have also explored superradiant transitions in systems with strong interactions [17–26], different statistics [27–33] and multimode cavities [34–40].

A recent experiment by Léonard et al. [39] has loaded a Bose condensate in two crossed beam cavities with the same frequency, as shown in Fig. 1a. The pumping beam has a wave vector $\mathbf{k}_p$ with a phase ϕ_p, and the two crossed cavity beams have wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ and phases ϕ_1 and ϕ_2, respectively. It is quite easy to see that under generic conditions, the system does not have the symmetry of choosing an arbitrary combination of these two cavity modes, because the interference of the pumping beam and different combinations of the two cavity beams will lead to different lattice structures, and consequently different energies for the atoms.

Nevertheless, this recent experiment reveals a surprise. They found that for pumping fields close to the critical value for the superradiant transition, the superradiant light can be emitted to any combination of these two cavity modes when the experiments were repeated under the same conditions. This implies that an additional $U(1)$ symmetry appears aside from the $U(1)$ symmetry breaking in the superradiant transition studied previously. The

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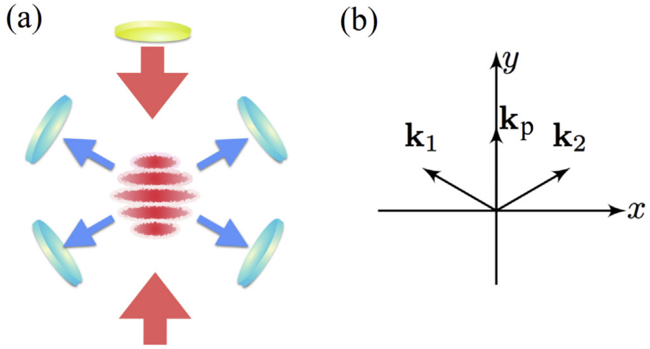

Fig. 1. (Color online) Schematic of the two crossed beam cavities (a) and the wave vectors of the pumping field and two cavity fields (b) in the experiment by Léonard et al. The experimental setup clearly satisfies the conditions (i) and (ii) we establish in the text.

main purpose of this paper is to point out that this additional $U(1)$ symmetry is actually an emergent symmetry in the vicinity of the superradiant transition, and we identify that this emergent symmetry requires the following conditions:

(i) The two cavity beams possess mirror symmetry with respect to the pumping beam;
(ii) The wave vectors of the three lasers satisfy $\mathbf{k}_\mathrm{p} = \mathbf{k}_1 + \mathbf{k}_2$;
(iii) The phases of the three lasers satisfy $\phi_\mathrm{p} - \phi_1 - \phi_2 = (l_\phi - 1/2)\pi$, where l_ϕ is an arbitrary integer;
(iv) The scattering between the two cavity modes is sufficiently weak so that it can be ignored.

Without loss of generality, the pumping beam can always be taken along the $\hat{y}$-direction, that is, $\mathbf{k}_\mathrm{p} = k_\mathrm{p}\hat{y}$. Thus, with conditions (i) and (ii), one can write $\mathbf{k}_1 = k_x\hat{x} + k_\mathrm{p}/2\hat{y}$ and $\mathbf{k}_2 = -k_x\hat{x} + k_\mathrm{p}/2\hat{y}$. In the experiment by Léonard et al. [39], the two cavity beams and the pumping beam have the same wave length and are aligned in 60-degree angle with respect to each other. Furthermore the beam phases are chosen to be $\phi_1 = \phi_2 = 0$ and $\phi_\mathrm{p} = \pi/2$. Thus the experimental set up is a special case that satisfies conditions (i), (ii) and (iii). In addition, condition (iv) is also well satisfied in the experiment. That is why the emergent $U(1)$ symmetry is observed there. Crucially, our demonstration of the emergent symmetry does not rely on the condition that the Bose gas forms a condensate. In other words, our findings apply to a thermal Bose gas as well as to a Bose condensate.

We should point out that the existence of this additional $U(1)$ symmetry has in fact been discussed earlier within a truncated five-mode model in Ref. [39]. It is important to emphasize the differences between the theoretical discussions made in Ref. [39] and the results of our work. In the former, an effective Hamiltonian was derived by restricting the atomic motion to five plane wave modes and a rotational invariance was shown to exist among these atomic motion modes and the cavity field modes. This rational invariance is then offered as an explanation for the $U(1)$ symmetry observed in the experiment. However, such a rotational invariance pertains only to this particular five-mode model and it exists regardless of whether the momentum matching condition (our condition (ii)) or the phase matching condition (our condition (iii)) is satisfied. In other words, the theory based on the five-mode model implies that the $U(1)$ symmetry exists even one of these two conditions is violated. In this regard, our theory predicts quite a different outcome from that in Ref. [39]. Finally, because of the truncation of the Hilbert space in this five-mode model, the theory based on this model cannot be applied to a normal gas above the critical temperature. As we explicitly show in our paper, the emergent $U(1)$ symmetry can also exist above the critical temperature.

The rest of the paper is organised as follows. In Section 2, we derive the model Hamiltonian used to describe the experimental system. The detailed analysis of the emergent $U(1)$ symmetry is given in Section 3. There we first present an intuitive physical picture for this symmetry and then develop a quantitative Ginzburg-Landau theory to explicitly demonstrate its existence under the necessary conditions we establish. In Section 4, we explore the consequence of violating the necessary conditions for the $U(1)$ symmetry. In particular we demonstrate that the superradiant transition in this system can also be driven to a first order one when the system is tuned away from the point having the emergent symmetry. A resulting phase diagram is obtained for the relevant tuning parameters. In Section 5, we study the polariton excitations of the cavity-BEC system and show the characteristic mode-softening prior to the onset of the superradiance transition. All our results are summarised in the concluding Section 6.

2. Model

In the system of the experiment by Léonard et al. [39], a Bose condensate is trapped at the intersection of a pumping beam and two identical high finesse optical cavities. The pumping beam along the $\hat{y}$-direction generates a one-dimensional optical lattice potential $V_{1D}(\mathbf{r}) = (\Omega_\mathrm{p}^2/\Delta_a)\cos^2(\mathbf{k}_\mathrm{p}\cdot\mathbf{r} + \phi_\mathrm{p})$, where Ω_p is the Rabi frequency and $\Delta_\mathrm{a} < 0$ is the atom-pump detuning. In the absence of cavity photons, the Hamiltonian for the atomic part is given by

$$\hat{H}_\mathrm{at} = \int d\mathbf{r}\hat{\psi}^\dagger(\mathbf{r})\hat{h}\hat{\psi}(\mathbf{r}) + \frac{1}{2}g_\mathrm{a}\int d\mathbf{r}\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})\hat{\psi}(\mathbf{r}), \quad (1)$$

where $\hat{h} = -\nabla^2/2m + V_{1D}(\mathbf{r})$ with m being the mass of the atom, $\hat{\psi}(\mathbf{r})$ is the atomic field operator and g_a is the interaction strength between atoms. The two cavities, labelled by the index $j = 1, 2$, are characterized by identical Rabi frequencies g and decay rates κ. The coupling between the cavity photons and the atoms is given by

$$\hat{H}_\mathrm{ph-at} = \frac{1}{2}\sqrt{UU_\mathrm{p}}\sum_{j=1,2}\left(\hat{a}_j + \hat{a}_j^\dagger\right)\int d\mathbf{r}f_j(\mathbf{r})\hat{\rho}(\mathbf{r}), \quad (2)$$

where $U_\mathrm{p} \equiv \Omega_\mathrm{p}^2/\Delta_\mathrm{a}$, $U \equiv g^2/\Delta_\mathrm{a}$, $\hat{a}_j$ annihilates a photon in cavity j, $\hat{\rho}(\mathbf{r}) \equiv \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})$ and

$$f_j(\mathbf{r}) \equiv 2\cos(\mathbf{k}_j\cdot\mathbf{r} + \phi_j)\cos(\mathbf{k}_\mathrm{p}\cdot\mathbf{r} + \phi_\mathrm{p}), \quad (3)$$

arises from the pump-cavity mode interference. Here we have already used condition (iv) to neglect the lattice potential from the interference of the two cavity fields. This can be justified because of the experimental condition $|U| \ll |U_\mathrm{p}|$ and the fact that the number of cavity photons is small in the vicinity of the superradiant transition. Under the rotating wave approximation, the Hamiltonian for the entire system can be written as

$$\hat{H} = -\sum_{j=1,2}\Delta_\mathrm{c}\hat{a}_j^\dagger\hat{a}_j + \hat{H}_\mathrm{at} + \hat{H}_\mathrm{ph-at}, \quad (4)$$

where $\Delta_\mathrm{c} < 0$ is the effective detuning between the cavity and the pumping field.

3. Emergent symmetry

3.1. An intuitive picture

We consider a rotation of the two cavity photon modes, i.e.

$$\hat{a}_1' = \cos\theta\,\hat{a}_1 + \sin\theta\,\hat{a}_2, \quad (5)$$
$$\hat{a}_2' = -\sin\theta\,\hat{a}_1 + \cos\theta\,\hat{a}_2. \quad (6)$$