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Group theory (GT) provides a rigorous framework for studying symmetries in various disciplines in physics ranging from quantum field theories and the standard model to fluid mechanics and chaos theory. To date, the application of such a powerful tool in optical physics remains limited. Over the past few years however, several quantum-inspired symmetry principles (such as parity-time invariance and supersymmetry) have been introduced in optics and photonics for the first time. Despite the intense activities in these new research directions, only few works utilized the power of group theory. Motivated by this status quo, here we present a brief overview of the application of GT in optics,deliberately choosing examples that illustrate the power of this tool in both continuous and discrete setups. We hope that this review will stimulate further research that exploits the full potential of GT for investigating various symmetry paradigms in optics, eventually leading to new photonic devices.
Keywords: Group theory, Helmholtz equation, Discrete optics, Parity-time (PT) symmetry

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1. INTRODUCTION

Symmetry principles play a crucial role in modern physics. Historically, one can trace interest in symmetry concepts to the early works on platonic solids by the ancient Greeks. In modern time, one of the first investigations that brought the notion of symmetry to the forefront of physical science was the discovery by Emmy Noether [1] that conservation laws and continuous symmetries are connected; e.g., the conservation of energy, linear and angular momenta are a direct outcome of temporal, spatial and rotational symmetries, respectively. Almost at the same time, the concept of gauge invariance was introduced by Herman Weyl in an attempt to unify gravity with electromagnetism. In quantum physics, the concepts of invariance under particle permutation (without or with a change of sign) led to the discovery of elementary particles classifications into bosons and fermions together with the associated particle statistics (Bose-Einstein or Fermi-Dirac). Despite these efforts, it was not until the seminal work of Wigner that group theory (the mathematical tool for studying symmetry) was formally employed in physics- particularly to study how symmetries of molecular configurations affect their vibrational spectra. Nowadays, group theory is an integral part of physics with applications ranging from quantum field theories, atomic and nuclear physics to quantum information and chaos theory to just mention few examples.

In optics, the tremendous progress over the past few decades have benefited a little from group theoretical techniques, except for a few notable examples (see Ref. [2] and references therein as well as Refs. [3–7]). Recently however, new research directions that exploits how quantum-inspired symmetries can be used to engineer novel optical structures have emerged. These include for instance PT symmetry (and non-Hermiticity in general) [8–38] and supersymmetry [39–50]. Despite recent intense activities in these fields, group theory remains largely outside the scope of these studies. In this article, we try to bridge this gap (at least partially) by presenting a brief review over some of the applications of group theory in continuous and discrete photonic systems in order to demonstrate its power with the hope that this will encourage further research in this largely unexplored area of research. For sake of clarity, we present more details on some of the technical mathematical terms used here in the Supplementary Data (online).

In general, systems admitting analytic solutions are rare, and when they exist, they have an underlying symmetry that can be exploited by group theory to obtain their solutions. The most important method for this pur-