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On the modified Minkowski functional minimization controller for uncertain systems with input and state constraints

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ABSTRACT

Two new stabilizing control schemes are presented for uncertain linear constrained systems on a controlled invariant set $\mathcal{C}$. The first scheme is an extension of the control law in Blanchini (1994), where the requirement that $\mathcal{C}$ is contractive in one step is relaxed. The second scheme aims to improve the performance by using a modified Minkowski functional concept. Proofs of recursive feasibility and robust asymptotic stability are provided. One numerical example with comparison to earlier solutions shows the main benefits of the proposed methods.

1. Introduction

Over the last decades, the regulation problem of a uncertain system with input and state constraints has been extensively studied. Especially in the discrete time case, different solutions are available as, for example, those based on min-max Model Predictive Control (MPC) (Bemporad, Morari, Dua, & Pistikopoulos, 2002; Camacho & Alba, 2013), or interpolating control (Nguyen, 2013; Rossiter, Kouvaritakis, & Bacic, 2004) or Minkowski functional minimization control (Blanchini, 1994; Blanchini & Miani, 2007).

In min-max MPC, a sequence of control actions is obtained which ensures the constraint satisfaction along the predicted trajectory of the plant for any possible uncertainty, and minimizes the worst case performance index of the predicted evolution of the plant. Solving this problem can be very demanding for large dimensional system and/or long prediction horizon, as they are NP hard (Lee & Yu, 1997). Thus the implementation of this type of control is limited to applications with relatively slow dynamics and small-scale processes.

Several attempts have been made in the literature to overcome the computational load problem of min-max MPC. In Bemporad, Borrelli, and Morari (2003), it was shown that the min-max MPC with a linear cost is equivalent to a multi-parametric linear program, where the state plays the role of a vector of parameters for the optimization problem. The solution is the so called explicit MPC where the control is a piecewise affine function of the state over a polyhedral partition of the state space, and the computational effort of the min-max MPC is moved off-line. However, for large dimensional systems and/or long prediction horizon the explicit solution may be very complex due to a high number of polyhedral cells. In Kothare, Balakrishnan, and Morari (1996), a new min-max MPC formulation was proposed that minimizes a quadratic cost function. At each time instant a *linear* state feedback law is obtained by solving a semi-definite programming problem involving linear matrix inequality constraints. In Wan and Kothare (2003) and Angeli, Casavola, Franzè, and Mosca (2008), an off-line ellipsoidal min-max MPC scheme is proposed by using a sequence of linear state feedback laws that correspond to a sequence of nested asymptotically stable invariant ellipsoids.

Interpolating control is an alternative approach which can be used to avoid the computational complexity problem of min-max MPC (Nguyen, 2013; Rossiter et al., 2004). The main idea is to employ a set of pre-defined controllers to overcome limitations of a single controller. However, the full control range is usually not exploited, since the control vector is not a decision variable in the optimization problem. Hence the time to regulate the plant to the origin is longer than necessary. Note that the approach in Cannon and Kouvaritakis (2005) has the same performance weakness as the interpolating control in Rossiter et al. (2004) and Nguyen (2013).

A Minkowski functional minimization control law is proposed in Blanchini (1994). The basic idea is to calculate the control action that minimizes the Minkowski functional of the one step ahead state prediction for a given $\lambda-$contractive set $\mathcal{C}$, $\lambda \in [0\ 1)$.

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At most two optimization problems of modest size are required to be solved at each time instant. The main weakness of this control law is that there is no tuning parameter in the case that the closed-loop system performance is poor, which is often the case for states near the origin (Blanchini & Miani, 2007). In Darup and Mönnigmann (2015), a predictive like control law is presented that can stabilize any state in an arbitrarily close inner approximation of a controlled invariant set.

Two novel constrained control laws are proposed in this paper. The first one is an extension of Blanchini (1994), where the requirement that $\mathcal{C}$ is contractive in one step is relaxed. We also point out that the algorithm in Blanchini (1994) may fail in this case. The second one aims to improve the performance by introducing a new definition of the modified Minkowski functional. This functional can be seen as a distance of a point from a set. At each time instant at most two optimization problems of modest size are required to be solved. Proofs of recursive feasibility and asymptotic stability are provided.

This paper is organized as follows. Section 2 is dedicated to the basic definitions and Section 3 to the Minkowski functional minimization control. In Section 4 results on the design of a novel stabilizing constrained controller are presented. One simulated example with comparison to earlier solutions is evaluated in Section 5 before drawing the conclusions in Section 6.

2. Problem formulation and preliminaries

2.1. Problem formulation

We consider the problem of regulating to the origin the following uncertain and/or time-varying discrete-time linear system

$$x(t+1) = A(t)x(t) + B(t)u(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the measurable state and the input, respectively. The system matrices $A(t) \in \mathbb{R}^{n\times n}$ and $B(t) \in \mathbb{R}^{n\times m}$ satisfy

$$\begin{cases} A(t) = \sum_{i=1}^{s} \alpha_i(t)A_i, \quad B(t) = \sum_{i=1}^{s} \alpha_i(t)B_i, \\ \sum_{i=1}^{s} \alpha_i(t) = 1, \quad \alpha_i(t) \geq 0, \forall i = 1, 2, \ldots, s \end{cases} \tag{2}$$

In almost all the practical applications, physical bounds on the state vector $x(t)$ and control vector $u(t)$ are present (Blanchini & Miani, 2007). Broadly speaking, the control law must be designed such that $x(t)$ of the closed-loop system is confined to a compact region, named the allowable state region, while $u(t)$ does not violate its constraints. Hence, the following is assumed

$$\begin{cases} x(t) \in X, \; X = \{x \in \mathbb{R}^n : F_x x \leq g_x\}, \\ u(t) \in U, \; U = \{u \in \mathbb{R}^m : F_u u \leq g_u\} \end{cases} \tag{3}$$

where $g_x > 0$ and $g_u > 0$. Element-wise inequalities are considered for simplicity, but the technique in the paper can be straightforwardly extended to more general mixed state/input constraints.

2.2. Basic definitions

Definition 1. Given two polytopes $\mathcal{P}$, $\mathcal{Q}$, their Minkowski sum, denoted as $\mathcal{P} \oplus \mathcal{Q}$, is the polytope

$$\mathcal{P} \oplus \mathcal{Q} = \{z | \; \exists (p, q) \in (\mathcal{P}, \mathcal{Q}) \text{ such that } z = p + q\} \tag{4}$$

The following lemma will be used for the recursive feasibility proof of the control scheme presented in this paper.

Lemma 1. *(Oks & Sharir, 2006) For a given polytope $\mathcal{P}$ and two scalars $\mu \geq 0$ and $\lambda \geq 0$, the following equality holds*

$$\mu\mathcal{P} \oplus \lambda\mathcal{P} = (\mu + \lambda)\mathcal{P} \tag{5}$$

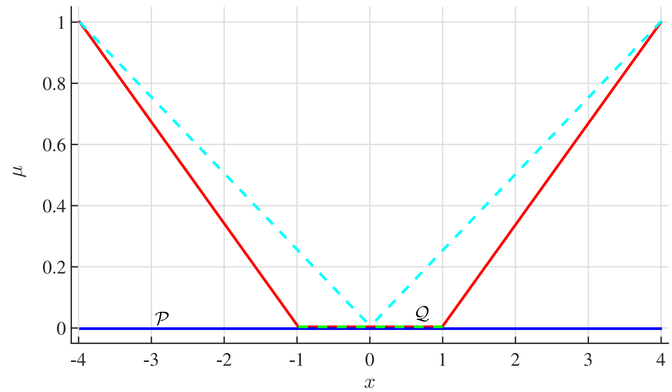

Fig. 1. Modified Minkowski functional (solid red), and Minkowski functional (dashed cyan) as a function of x, and the sets $\mathcal{P}$ (solid blue) and $\mathcal{Q}$ (dashed green). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Definition 2. (Blanchini & Miani, 2007) Given a polytope $\mathcal{P}$, containing the origin in its interior. For a given $x \in \mathcal{P}$, the Minkowski functional of $\mathcal{P}$ is defined as,

$$\Psi_{\mathcal{P}}(x) = \min_{\mu}\{\mu \geq 0 : x \in \mu\mathcal{P}\} \tag{6}$$

The function $\Psi_{\mathcal{P}}(x)$ is convex, positively homogeneous of order one, i.e. $\Psi_{\mathcal{P}}(\epsilon x) = \epsilon\Psi_{\mathcal{P}}(x)$ for any $\epsilon \geq 0$. It can be seen $\Psi_{\mathcal{P}}(x)$ as a measure from x to the origin.

Definition 3. Given two polytopes $\mathcal{P}$, $\mathcal{Q}$, $0 \in \mathcal{Q} \subseteq \mathcal{P}$. For a given $x \in \mathcal{P}$, the modified Minkowski functional of $\mathcal{P}$ with respect to $\mathcal{Q}$ is defined as

$$\Psi_{\mathcal{P}\to\mathcal{Q}}(x) = \min_{\mu,r}\{\mu\} \text{ s.t. } \begin{cases} 0 \leq \mu \leq 1 \\ x - r \in \mu\mathcal{P} \\ r \in (1-\mu)\mathcal{Q} \end{cases} \tag{7}$$

Note that r is a vector in (7).

Remark 1. If $\mathcal{Q}$ is the origin, then r is a zero vector in (7). Hence (7) coincides with (6). Therefore the Minkowski functional is a particular case of the modified Minkowski functional.

The modified Minkowski functional can be seen as a distance from x to the set $\mathcal{Q}$ *normalized* by the set $\mathcal{P}$. Note that

- $\mu = 1$ for all x on the boundary of $\mathcal{P}$.
- $0 < \mu \leq 1$ for all $x \in P\backslash Q$.
- $\mu = 0$, for all $x \in \mathcal{Q}$.

For example, consider the following polytopes $\mathcal{P} \subset \mathbb{R}$ and $\mathcal{Q} \subset \mathbb{R}$,

$$\begin{aligned} \mathcal{P} &= \{x \in \mathbb{R} : -4 \leq x \leq 4\}, \\ \mathcal{Q} &= \{x \in \mathbb{R} : -1 \leq x \leq 1\} \end{aligned} \tag{8}$$

In this case, μ given from (7) can be explicitly calculated as a piecewise affine function of x

$$\mu = \begin{cases} \frac{1}{3}x - \frac{1}{3}, & \text{if } x \geq 1, \\ 0, & \text{if } -1 \leq x \leq 1, \\ -\frac{1}{3}x - \frac{1}{3}, & \text{if } x \leq -1 \end{cases} \tag{9}$$

Fig. 1 presents the modified Mikowski functional (solid red) and the Minkowski functional (dashed cyan) as a function of x as well as the sets $\mathcal{P}$ (solid blue) and $\mathcal{Q}$ (dashed green). It can be verified in Fig. 1 that the modified Minkowski functional is always equal to or smaller than the Minkowski functional, since that latter one is a particular case of the former with $r = 0$.