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Hierarchical distributed ADMM for predictive control with applications in power networks[☆,☆☆]

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A B S T R A C T

In this paper, we investigate optimal control and operation of a network of linear, physically decoupled systems with a coupling in the objective function. To deal with the corresponding distributed control problem, we propose a new Model Predictive Control (MPC) scheme based on the Alternating Direction Method of Multipliers (ADMM). In particular, we thoroughly investigate the flexibility of the proposed hierarchical distributed MPC algorithm with respect to both its plug-and-play capability and changes in the (local) system dynamics and objective functions at runtime. Moreover, we show linear scalability in the number of subsystems. The efficacy of the distributed optimization algorithm embedded in MPC is illustrated on three battery scheduling problems arising from the predictive control of residential microgrid electricity networks.

1. Introduction

Optimal control and operation of large-scale cyber-physical systems presents numerous challenges, among them: the curse of dimensionality; security concerns due to sharing of sensitive information between subsystems; and the desirability of adding new subsystems without requiring a coordinating agent to possess detailed information about such subsystems, so-called "plug-and-play" functionality. Distributed optimization algorithms used in this context, such as dual decomposition, date back to the early 1960s, (Benders, 1962; Dantzig & Wolfe, 1960; Everett, 1963). These algorithms split the task of minimizing a single, large optimization problem into smaller independent subproblems, which are then either solved in parallel or sequentially. Progress on distributed optimization algorithms led to the alternating direction method of multipliers (ADMM), (Fortin & Glowinski, 1983; Gabay, 1983; Gabay & Mercier, 1976), which, along with other distributed optimization algorithms, are extensively studied in Bertsekas and Tsitsiklis (1989) and Tsitsiklis (1984). The rediscovery and popularity of distributed optimization algorithms in recent years is evidenced by the highly cited recent paper on ADMM by Boyd, Parikh, Chu, Peleato, and Eckstein (2011), and the development of new methods such as those proposed in Houska, Frasch, and Diehl (2016), Jakovetić, Xavier, and Moura (2014) and Kia, Cortés, and Martínez (2015), and the references therein.

In this paper we propose a hierarchical distributed Model Predictive Control (MPC) algorithm applicable to physically decoupled subsystems with a coupling in the objective function (see Grüne & Pannek, 2017 and the references therein for an introduction to MPC). The MPC scheme uses an ADMM algorithm based on Boyd et al. (2011, Chapter 7) in the optimization step but in a tailored version. In particular, the subsystems communicate with a central entity rather than using a neighbor-to-neighbor communication structure. By using ADMM in the hierarchical distributed MPC algorithm, the proposed predictive control scheme is flexible

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with respect to both the system dynamics and the objective function. This means that the system dynamics of the subsystems as well as the objective function can be changed online in the MPC closed loop operation. Additionally, it scales well with the number of subsystems, thereby addressing the aforementioned computational, security, and plug-and-play functionality challenges. While the proposed MPC scheme has a similar communication structure as the algorithm defined in the earlier work (Braun, Grüne, Kellett, Weller, & Worthmann, 2016), using ADMM offers significantly more flexibility with respect to the objective function and the system dynamics.

As an application of our MPC algorithm we consider three different problems related to the optimal scheduling of energy storage in a residential-scale electricity network. The first application is taken from Worthmann, Kellett, Braun, Grüne, and Weller (2015), where we addressed the problem of smoothing the energy demand from a group of residences equipped with energy storage. The second application further extends this setting by considering additional time-varying tube constraints that enable a grid operator to steer demand within reasonable limits. Such an approach gives energy providers flexibility when making operational decisions on both the quantity and type of external generation resources to be employed. The third application enables computation and/or implementation of the maximal islanding time for a microgrid. In this context, the proposed ADMM variant enables a grid operator to forecast the maximal duration of grid disconnection, while maintaining sufficient local supply to meet demand. Implemented in a predictive control context, the microgrid can collectively store energy prior to the disconnection time, so as to maximize the time it can operate independently.

Distributed MPC for linear systems has been extensively studied by Venkat and his coauthors and significant contributions in this field have been obtained in Stewart, Venkat, Rawlings, Wright, and Pannocchia (2010), Venkat (2006), Venkat, Hiskens, Rawlings, and Wright (2006) and Venkat, Hiskens, Rawlings, and Wright (2008). For coupled linear systems, coupled in the objective function as well as in the dynamics, distributed MPC algorithms have been proposed, which allow for local optimization and guarantee closed loop stability with respect to a given setpoint. In contrast to the work of Venkat et al., we restrict our attention to a coupling in the objective function. This allows us to concentrate on scalability and flexibility with respect to the network structure and scalability with respect to the communication structure of our proposed distributed MPC algorithm. Moreover, it enables us to keep the communication structure independent of the system dynamics, in contrast to the work by Venkat et al. To ensure closed loop stability Venkat et al. concentrate on quadratic cost functions which are strongly convex in the input u. In this paper we allow more general convex (i.e., not necessarily strongly convex) cost functions in the context of reference tracking instead of stability considerations. In this more general setting, we show that the solution of the distributed optimization problem provides the same open loop costs as a centralized solution of the optimization problem. Moreover, our approach uses ADMM to split the coupled optimization into local optimization problems, whereas the optimization strategies in the works of Venkat et al. rely on the exchange of locally updated inputs u in every iteration.

ADMM embedded in a receding horizon scheme has also been proposed in Conte, Summers, Zeilinger, Morari, and Jones (2012) and Scott and Thiébaux (2015) in the context of microgrids. As distinct from the present paper, however, neither Conte et al. (2012) nor Scott and Thiébaux (2015) implement ADMM as a hierarchical algorithm. Rather, both Conte et al. (2012) and Scott and Thiébaux (2015) use neighbor-to-neighbor communication instead of communication with a central entity. Consequently, communication structure

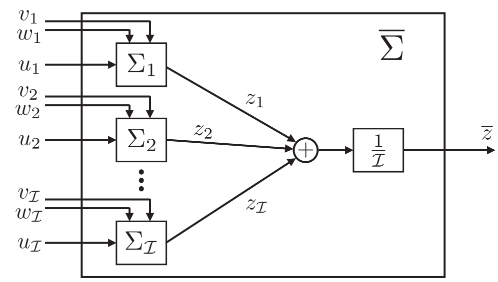

Fig. 1. Visualization of the individual systems Σ_i and the overall system $\overline{\Sigma}$.

and algorithm flexibility are not the focus of these papers. In Atzeni, Ordóñez, Scutari, Palomar, and Fonollosa (2013), Kraning, Chu, Lavaei, and Boyd (2013), and Le Floch, Belletti, Saxena, Bayen, and Moura (2015), related distributed optimization algorithms are used in the context of smart grids. None of these algorithms, however, is embedded in an MPC scheme. In Atzeni et al. (2013) and Kraning et al. (2013) proximal algorithms are used to solve optimization problems for the optimal operation of a microgrid, and in Le Floch et al. (2015) dual decomposition algorithms are proposed to optimally charge a fleet of electric vehicles.

The paper is structured as follows: In Section 2 we formulate an optimal control problem in the form of a nonlinear optimization problem (NLP) for linear time-varying discrete-time control systems coupled through a set of variables in the objective function. In Section 3 we introduce a hierarchical distributed optimization algorithm and embed it in a receding horizon framework. In Section 4 we recall the system dynamics of an electricity network satisfying the assumptions of the dynamics introduced in Section 2. The flexibility of the hierarchical distributed optimization algorithm with respect to the objective in the context of model predictive control is demonstrated in Section 5 based on the example of the microgrid. The paper concludes in Section 6.

2. Optimal control formulation for a network of linear systems

We consider a network of $\mathcal{I} \in \mathbb{N}$ linear time-varying discrete-time systems

$$\Sigma_i : \begin{cases} x_i(k+1) = A_i(k)x_i(k) + B_i(k)u_i(k) + v_i(k) \\ z_i(k) = E_i(k)x_i(k) + F_i(k)u_i(k) + w_i(k) \end{cases} \tag{1}$$

where $x_i \in \mathbb{X}_i \subset \mathbb{R}^{n_i}$, $u_i \in \mathbb{U}_i \subset \mathbb{R}^{m_i}$, $z_i \in \mathbb{R}^p$ are the state, input, and coupling variables, respectively, $(v_i(k))_{k\in\mathbb{N}} \subset \mathbb{R}^{n_i}$ and $(w_i(k))_{k\in\mathbb{N}} \subset \mathbb{R}^p$ are known exogenous signals, and $A_i(k) \in \mathbb{R}^{n_i\times n_i}$, $B_i(k) \in \mathbb{R}^{n_i\times m_i}$, $E_i(k) \in \mathbb{R}^{p\times n_i}$, $F_i(k) \in \mathbb{R}^{p\times m_i}$ are time-dependent matrices defining the system dynamics for all $k \in \mathbb{N}$ and for all $i \in \mathbb{N}_\mathcal{I} := \{1, \ldots, \mathcal{I}\}$.

As shown in Fig. 1, systems Σ_i are coupled through the variables z_i, $i \in \mathbb{N}_\mathcal{I}$, leading to the overall system dynamics

$$\overline{\Sigma} : \begin{cases} x(k+1) = A(k)x(k) + B(k)u(k) + v(k) \\ \overline{z}(k) = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} (E_i(k)x_i(k) + F_i(k)u_i(k) + w_i(k)) \end{cases}$$

with

$$x = (x_1^T, \ldots, x_\mathcal{I}^T)^T, \quad u = (u_1^T, \ldots, u_\mathcal{I}^T)^T,$$
$$v = (v_1^T, \ldots, v_\mathcal{I}^T)^T, \quad w = (w_1^T, \ldots, w_\mathcal{I}^T)^T,$$

and the averaged coupling variable $\overline{z} = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} z_i$. The definition of matrices $A(k)$, $B(k)$ follow immediately from the definition of the individual dynamics Σ_i. Additionally we use the notation $\mathbb{X} = \mathbb{X}_1 \times \cdots \times \mathbb{X}_\mathcal{I}$ and $\mathbb{U} = \mathbb{U}_1 \times \cdots \times \mathbb{U}_\mathcal{I}$ to rewrite the state and input constraints. Throughout this paper we assume that the states x_i are known or at least observable.