IFAC Journal of Systems and Control 2 (2017) 12–17

Contents lists available at ScienceDirect

IFAC Journal of Systems and Control

journal homepage: www.elsevier.com/locate/ifacsc

Invariant sliding domains for constrained linear receding horizon tracking control

Andreas Hansen[a,*], Yutong Li[b], J. Karl Hedrick[a]

[a] Department of Mechanical Engineering, University of California, Berkeley, CA 94720, USA
[b] State Key Laboratory of Automotive Safety and Energy, Tsinghua University, Beijing 100084, China

ARTICLE INFO

Article history:
Received 14 May 2017
Revised 26 September 2017
Accepted 4 November 2017
Available online 6 November 2017

Keywords:
Sliding control
Model predictive control
Constrained optimization
Setpoint tracking
Linear systems

ABSTRACT

A classical approach for guaranteeing persistent feasibility of model predictive controllers during setpoint changes adds an artificial reference variable, whereby allowing for reference offset at a cost specified by an additional term in the cost function. Typically, the classical approach employs a linear quadratic regulator parameterized by the artificial reference as a terminal control law and hence requires invariant set computations in an augmented state/reference space. This paper develops a receding horizon sliding control technique for constrained linear setpoint tracking. By exploiting the flatness property of sliding hyperplanes, the artificial reference can be eliminated from the control scheme and the terminal invariant set is contained in the original dimensions of the state space only. The proposed dual mode receding horizon control design approach is proven to maintain persistent feasibility and stability.

1. Introduction

The key benefit of model predictive control (MPC) over other control techniques is constraint satisfaction, but it comes at the expense of a challenging persistent feasibility and stability analysis. For regulation problems, among other approaches, there is mature theory addressing the feasibility/stability issue by adding a suitable terminal invariant set constraint and a terminal cost term to the MPC scheme (Borrelli, Bemporad, & Morari, 2017). However, when target setpoint changes occur, MPC regulation schemes can still become infeasible.

The issue of changing setpoints was addressed in the literature by several authors (Dughman & Rossiter, 2015). Predictive reference management for changing references is presented in Bemporad, Casavola, and Mosca (1997). A different approach is taken in Simon, Löfberg, and Glad (2014), where an optimization variable for terminal constraint scaling is added in order to avoid infeasibility. Moreover, in a series of publications (Ferramosca, Limon, Alvarado, Alamo, & Camacho, 2009; Ferramosca et al., 2011; Limon, Alvarado, Alamo, & Camacho, 2008; Limon, Ferramosca, Alvarado, Alamo, & Camacho, 2009) the authors develop an MPC scheme that allows changing setpoints by adding an additional optimization variable which has the interpretation of an artificial reference. Moreover, an additional term in the cost function that penalizes the deviation between the artificial reference and the actual reference is added. An extended terminal invariant set constraint that is formulated in terms of the augmented state including the artificial reference guarantees persistent feasibility. The authors also prove asymptotic stability with respect to feasible desired setpoints and the local optimality property (Ferramosca et al., 2011).

Extending the state vector with the reference state is necessary in previous works (Ferramosca et al., 2009; Ferramosca et al., 2011; Limon et al., 2008; Limon et al., 2009) because terminal sets corresponding to different setpoints would overlap otherwise, which introduces ambiguity. This paper exploits the flatness property of sliding hyperplanes to eliminate the artificial reference, which results in an invariant set solely contained in the original state-space dimensions. Having a lower dimensional state space can reduce the computational burden of invariant set computations in some applications and generally reduces the number of constraints necessary to describe the set.

The novel invariant set for tracking introduced in this paper is then incorporated in a receding horizon sliding control scheme (RHSC). RHSC combines design principles from MPC and sliding control (Hansen & Hedrick, 2015). In a receding horizon fashion, the controller is designed to minimize the deviation of the system's state to sliding surfaces/manifolds (Utkin, Guldner, & Shi, 2009). Recent works indicate that this alternative shaping of the cost function can be beneficial in some practical control applications, see e. g. Li, Hansen, Hedrick, and Zhang (2017) and

* Corresponding author.
E-mail addresses: a.hansen@berkeley.edu (A. Hansen), liyutong12@mails.tsinghua.edu.cn (Y. Li).

https://doi.org/10.1016/j.ifacsc.2017.11.001
2468-6018/

Sudhakar, Hansen, and Hedrick (2016). The controller presented in this paper maintains all of the key properties of the MPC from Limon et al. (2008, 2009) and Ferramosca et al. (2009, 2011) and allows for eliminating the artificial reference from the control scheme resulting in a slightly reduced number of optimization variables.

The remainder of this paper is organized as follows. In Section 2 the concept of invariant sliding domains, i. e. invariant sets induced by sliding control laws, is derived. Subsequently, in Section 3 the invariant sliding domains from Section 2 are included in a receding horizon control framework and we provide feasibility and stability proofs of the resulting schemes. Section 4 contains an illustrative example and Section 5 draws conclusions from this work.

2. Invariant sliding domains

It is well known that classical sliding control induces invariant sets in state space (Slotine & Li, 1991; Utkin et al., 2009). This section formalizes these invariant sets in the presence of constraints. First, we present the considered control scenarios. Then, we derive terminal sliding control laws. Finally, we define two invariant sets of different complexity for receding horizon control applications.

2.1. Preliminaries

Consider square MIMO systems in state space form, where the state is denoted $\boldsymbol{x}(k) \in \mathbb{R}^n$ and $\boldsymbol{u}(k), \boldsymbol{y}(k) \in \mathbb{R}^m$ are the input and output at time-step k. Furthermore, we assume an exact prediction model of the form

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{B}\boldsymbol{u}_k, \tag{1}$$

$$\boldsymbol{y}_k = \boldsymbol{C}\boldsymbol{x}_k. \tag{2}$$

We use standard notation, where the time-step is indicated by the subscript in order to express that Eqs. (1) and (2) yield model-based predictions of the actual system signals. The system matrices are $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, $\boldsymbol{B} \in \mathbb{R}^{n\times m}$, and $\boldsymbol{C} \in \mathbb{R}^{m\times n}$. Furthermore, the state and input are constrained by

$$\boldsymbol{x} \in \mathcal{X} \subseteq \mathbb{R}^n, \tag{3}$$

$$\boldsymbol{u} \in \mathcal{U} \subseteq \mathbb{R}^m. \tag{4}$$

We restrict the sets $\mathcal{X}$ and $\mathcal{U}$ to have the origin in their interior and to be polyhedral. Additionally, we make the following assumption on the system.

Assumption 1. The system (1), (2)

- has relative degree $(d_1, \ldots, d_m)$;
- is minimum-phase, i. e. the system only has transmission zeros strictly inside the unit circle of the complex plane.

This paper uses the notion of relative degree from Isidori (1995) that is explicitly formalized for linear systems in Kraev, Rogovskii, and Fomichev (2014, Definition 1). Note that the minimum-phase assumption implies stabilizability and detectability of (1), (2) (Davison, 1983). The control goal is to let the system track a piecewise constant desired output signal, $\boldsymbol{r}_k$, i. e. we desire the tracking error,

$$\boldsymbol{e}_k = \boldsymbol{y}_k - \boldsymbol{r}_k, \tag{5}$$

to go to zero while accounting for the system constraints.

2.2. Terminal sliding controller

Sliding control is utilized for obtaining a terminal state feedback control law. Specifically, for each of the m output error components we define a Schur polynomial (Kraus, Anderson, & Mansour, 1988) encoding the desired eigenvalues of the output error dynamics. The degree of these error dynamics is $l_i := d_i - 1,\ i = 1, \ldots, m$. For invariant set computations we replace $\boldsymbol{r}_k$ in (5) by an artificial reference variable, $\tilde{\boldsymbol{r}}$, and form $\tilde{\boldsymbol{e}}_k = \boldsymbol{y}_k - \tilde{\boldsymbol{r}}$ (Limon et al., 2008). With abuse of notation we introduce the one-step ahead operator z and define a sliding variable by following the standard approach from Isidori (1995) and Sira-Ramírez (1991) as

$$\boldsymbol{s}_k := \begin{bmatrix} \left(\sum_{j=0}^{l_1} \alpha_{1,j} z^j\right) \tilde{e}_{1,k} \\ \vdots \\ \left(\sum_{j=0}^{l_m} \alpha_{m,j} z^j\right) \tilde{e}_{m,k} \end{bmatrix} \tag{6}$$

$$= \begin{bmatrix} \alpha_{1,0}\tilde{e}_{1,k} + \cdots + \alpha_{1,l_1}\tilde{e}_{1,k+l_1} \\ \vdots \\ \alpha_{m,0}\tilde{e}_{m,k} + \cdots + \alpha_{m,l_m}\tilde{e}_{m,k+l_m} \end{bmatrix}. \tag{6}$$

There are two design restrictions that we enforce. Firstly, for stability reasons the polynomials $\sum_{j=0}^{l_i} \alpha_{i,j} z^j$ with $i = 1, \ldots, m$ are required to be chosen such that all roots are strictly inside of the unit circle. Secondly, we require $\alpha_{i,l_i} \neq 0,\ i = 1, \ldots, m$. Farther, without loss of generality we normalize such that $\alpha_{i,l_i} = 1,\ i = 1, \ldots, m$.

By recursively substituting the system model (1), (2) in (6) we arrive at the following representation for $\boldsymbol{s}_k$,

$$\boldsymbol{s}_k = \begin{bmatrix} \boldsymbol{c}_1^\top \sum_{j=0}^{l_1} \alpha_{1,j}\boldsymbol{A}^j \\ \vdots \\ \boldsymbol{c}_m^\top \sum_{j=0}^{l_m} \alpha_{m,j}\boldsymbol{A}^j \end{bmatrix} \boldsymbol{x}_k - \begin{bmatrix} \sum_{j=0}^{l_1} \alpha_{1,j} & & \\ & \ddots & \\ & & \sum_{j=0}^{l_m} \alpha_{m,j} \end{bmatrix} \tilde{\boldsymbol{r}}$$
$$=: \boldsymbol{G}\boldsymbol{x}_k - \boldsymbol{H}\tilde{\boldsymbol{r}}. \tag{7}$$

The form of $\boldsymbol{s}_k$ is standard, compare Spurgeon (1992), with an added constant involving the reference to account for reference tracking rather than regulation to the origin. In (7), all off-diagonal entries of $\boldsymbol{H}$ are zero and $\boldsymbol{c}_1^\top, \ldots, \boldsymbol{c}_m^\top$ denote the rows of $\boldsymbol{C}$. It is important to note that the specified design restrictions enforce $\boldsymbol{c}_i^\top \boldsymbol{A}^j \boldsymbol{B} = \boldsymbol{0}^{1\times m},\ j = 0, \ldots, l_i,\ i = 1, \ldots, m$ and hence these terms do not feature in (7).

Given a certain reference $\tilde{\boldsymbol{r}}$, from (7) it is easy to interpret $\{\boldsymbol{x} : \boldsymbol{s} = \boldsymbol{0}^{m\times 1}\}$ as the intersection of m hyperplanes in state space. On this intersection, the specified desired error dynamics $\alpha_{i,0}\tilde{e}_{i,k} + \cdots + \alpha_{i,l_i}\tilde{e}_{i,k+l_i} = 0,\ i = 1, \ldots, m$ hold. For $\{\boldsymbol{x} : \boldsymbol{s} \neq \boldsymbol{0}^{m\times 1}\}$, the components s_i are a measure for the distance of $\boldsymbol{x}$ to the desired manifolds $\{\boldsymbol{x}^* : \boldsymbol{g}_i^\top \boldsymbol{x}^* - H_{i,i}\tilde{r}_i = 0\}$ which is directly quantified by $s_i / \parallel \boldsymbol{g}_i \|_2,\ i = 1, \ldots, m$, where $\boldsymbol{g}_i^\top$ is the ith row of $\boldsymbol{G}$.

The so-called equivalent control law (Spurgeon, 1992; Utkin et al., 2009), that enforces the state to be on the sliding manifold in the subsequent time-step is found by setting $\boldsymbol{s}_{k+1} = \boldsymbol{0}^{m\times 1}$. Using (7) we find

$$\boldsymbol{u}_k^{\text{eq}} = -(\boldsymbol{G}\boldsymbol{B})^{-1}(\boldsymbol{G}\boldsymbol{A}\boldsymbol{x}_k - \boldsymbol{H}\tilde{\boldsymbol{r}}) =: \boldsymbol{K}\boldsymbol{x}_k + \boldsymbol{L}\tilde{\boldsymbol{r}}, \tag{8}$$

where existence of $(\boldsymbol{G}\boldsymbol{B})^{-1}$ is ensured by Assumption 1 (Kraev et al., 2014).