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The modified optimal velocity model: stability analyses and design guidelines☆

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ABSTRACT

Reaction delays are important in determining the qualitative dynamical properties of a platoon of vehicles traveling on a straight road. In this paper, we investigate the impact of delayed feedback on the dynamics of the Modified Optimal Velocity Model (MOVM). Specifically, we analyze the MOVM in three regimes – no delay, small delay and arbitrary delay. In the absence of reaction delays, we show that the MOVM is locally stable. For small delays, we then derive a sufficient condition for the MOVM to be locally stable. Next, for an arbitrary delay, we derive the necessary and sufficient condition for the local stability of the MOVM. We show that the traffic flow transits from the locally stable to the locally unstable regime via a Hopf bifurcation. We also derive the necessary and sufficient condition for non-oscillatory convergence and characterize the rate of convergence of the MOVM. These conditions help ensure smooth traffic flow, good ride quality and quick equilibration to the uniform flow. Further, since a Hopf bifurcation results in the emergence of limit cycles, we provide an analytical framework to characterize the type of the Hopf bifurcation and the asymptotic orbital stability of the resulting non-linear oscillations. Finally, we corroborate our analyses using stability charts, bifurcation diagrams, numerical computations and simulations conducted using MATLAB.

1. Introduction

Intelligent transportation systems constitute a substantial theme of discussion on futuristic smart cities. In this context, self-driven vehicles are a prospective solution to address traffic issues such as resource utilization and commute delays; (see Rajamani, 2012, Section 5.2, van den Berg and Verhoef, 2016; Greengard, 2015; Vahidi and Eskandarian, 2003 and references therein). To ensure that these objectives are met, in addition to ensuring human safety, the design of control algorithms for these vehicles becomes important. To that end, it is imperative to have an in-depth understanding of human behavior and vehicular dynamics. This has led to the development and study of a class of dynamical models known as the car-following models (Bando, Hasebe, Nakanishi, & Nakayama, 1998; Chowdhury, Santen, & Schadschneider, 2000; Gazis, Herman, & Rothery, 1961; Helbing, 2001; Kamath, Jagannathan, & Raina, 2015; Orosz & Stépán, 2006).

Feedback delays play an important role in determining the qualitative behavior of dynamical systems (Hale & Lunel, 2011). In particular, these delays are known to destabilize the system and induce oscillatory behavior (Kamath et al., 2015; Sipahi & Niculescu, 2006). In the context of human-driven vehicles, predominant components of the reaction delay are psychological and mechanical in nature (Sipahi & Niculescu, 2006). In contrast, delays in self-driven vehicles arise due to sensing, communication, signal processing and actuation, and are envisioned to be smaller than human reaction delays (Kesting & Treiber, 2008).

In this paper, we investigate the impact of delayed feedback on the qualitative dynamical properties of a platoon of vehicles traveling on a straight road. Specifically, we consider each vehicle's dynamics to be modeled by the Modified Optimal Velocity Model (MOVM) (Kamath et al., 2015). Motivated by the wide range of values assumed by reaction delays in various scenarios, we analyze the MOVM in three regimes; namely, (*i*) no delay, (*ii*) small delay and (*iii*) arbitrary delay. In the absence of delays, we show that the MOVM is locally stable. When the delays are rather small, as in the case of self-driven vehicles, we derive a sufficient condition for the local stability of the MOVM using a suitable approximation. For the arbitrary-delay regime, we analytically characterize the region of local stability for the MOVM.

In the context of transportation networks, two additional properties are of practical importance; namely, ride quality (lack of

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jerky vehicular motion) and the time taken by the platoon to attain the desired equilibrium when perturbed. Mathematically, these translate to studying the non-oscillatory property of the MOVM's solutions and the rate of their convergence to the desired equilibrium. In this paper, we also characterize these properties for the MOVM.

In the context of human-driven vehicles, model parameters generally correspond to human behavior, and hence cannot be "tuned" or "controlled." However, our work enhances phenomenological insight into the emergence and evolution of traffic congestion. For example, a peculiar phenomenon known as the "phantom jam" is observed on highways (Chowdhury et al., 2000; Helbing, 2001). Therein, a congestion wave emerges seemingly out of nowhere and propagates up the highway from the point of its origin. Such an oscillatory behavior in the traffic flow has typically been attributed to a change in the driver's sensitivity, such as a sudden deceleration; for details, see Chowdhury et al. (2000) and Helbing (2001). In general, feedback delays are known to induce oscillations in state variables of dynamical systems (Kamath et al., 2015; Sipahi & Niculescu, 2006). Since the MOVM explicitly incorporates feedback delays, and relative velocities and headways constitute state variables of the MOVM, our work provides a theoretical basis for understanding the emergence and evolution of oscillatory phenomena such as "phantom jams." In particular, our work serves to highlight the possible role of reaction delays in the emergence of oscillatory phenomena in traffic flows. More generally, our results reveal an important observation: the traffic flow may transit into instability due to an appropriate variation in *any* subset of model parameters. To capture this complex dependence of stability on various parameters, we introduce an exogenous, non-dimensional parameter in our dynamical model. We then analyze the behavior of the resulting system as the exogenous parameter is pushed just beyond the stability boundary. We show that nonlinear oscillations, termed *limit cycles*, emerge in the traffic flow due to a *Hopf bifurcation*.

In the context of self-driven vehicles, reaction delays are expected to be smaller than their human counterparts (Kesting & Treiber, 2008). Hence, it would be realistically possible to achieve smaller equilibrium headways (Rajamani, 2012, Section 5.2). This would, in turn, vastly improve resource utilization without compromising safety (Greengard, 2015). In this paper, based on our theoretical analyses, we provide some design guidelines to appropriately tune the parameters of the so-called "upper longitudinal control algorithm" (Rajamani, 2012, Section 5.2). Mathematically, our analytical findings highlight the quantitative impact of delayed feedback on the design of control algorithms for self-driven vehicles. Specifically, our design guidelines take into consideration various aspects of the longitudinal control algorithm such as stability, good ride quality and fast convergence of the traffic to the uniform flow. In the event that the traffic flow does lose stability, our design guidelines help tune the model parameters with an aim of reducing the amplitude and angular velocity of the resultant limit cycles.

1.1. Related work on car-following models

The motivation for our paper comes from the key idea behind the Optimal Velocity Model (OVM) proposed by Bando *et al.* in Bando, Hasebe, Nakayama, Shibata, and Sukiyama (1995) for a platoon of vehicles on a circular loop. However, the model considered therein was devoid of reaction delays. Thus, a new model was proposed in Bando et al. (1998) to account for the drivers' delays. Therein, the authors also claimed that these delays were not central to capturing the dynamics of the system. In response, Davis showed via numerical computations that reaction delays indeed play an important part in determining the qualitative behavior of the OVM Davis (2002). This led to a further modification to the OVM in Davis (2003). However, this too did not account for the delay arising due to a vehicle's own velocity. It was shown in Gasser, Sirito, and Werner (2004) that the OVM without delays loses local stability via a Hopf bifurcation. For the OVM with delays, Orosz, Krauskopf, and Wilson (2005) performed an initial numerical study of the bifurcation phenomenon before supplying an analytical proof in Orosz and Stépán (2006).

While a control-theoretic treatment of car-following models has been widely studied (see Bekey, Burnham, and Seo, 1977; Dey et al., 2016; Li et al., 2017 and references therein), the thematic issue on "Traffic jams: dynamics and control" (Orosz, Wilson, & Stépán, 2010) highlights the growing interest in a synergized control-theoretic and dynamical systems viewpoint of transportation networks. A recent exposition of linear stability analysis in the context of car-following models can be found in Wilson and Ward (2011).

From a vehicular dynamics perspective, most upper longitudinal controllers in the literature assume the lower controller's dynamics to be well modeled by a first-order control system, in order to capture the delay lag (Rajamani, 2012, Section 5.3). The upper longitudinal controllers are then designed to maintain either constant velocity, spacing or time gap; for details, see Rajamani and Zhu (2002) and the references therein. Specifically, it was shown in Rajamani and Zhu (2002) that synchronization with the lead vehicle is possible by using information only from the vehicle directly ahead. This reduces implementation complexity, and does not mandate vehicles to be installed with communication devices.

However, in the context of autonomous vehicles, communication systems are required to exchange various system states required for the control action. This information is used either for distributed control (Rajamani & Zhu, 2002) or coordinated control (Qu, Wang, & Hull, 2008) of vehicles. Formation control (Anderson, Sun, Sugie, Azuma, & Sakurama, 2017; Chavan, Belur, Chakraborty, & Manjunath, 2015) and platoon stabilities (Summers, Yu, Dasgupta, & Anderson, 2011) have also been studied considering information flow among the vehicles. However, these works do not consider the effect of delays in relaying the required information. In contrast, when latency increases due to randomness in the communication environment, strategies have been developed to make use of only on-board sensors with minimal degradation in performance (Ploeg, Semsar-Kazerooni, Lijster, van de Wouw, & Nijmeijer, 2015). For an extensive review, see Dey et al. (2016). Usage of communication systems is also known to mitigate phantom jams (Won, Park, & Son, 2016). It may be noted that, for our scenario of straight road with a single lane, the formation control problem subsumes the problem of stabilizing a platoon. Thus, our work can also be thought of as a formation control problem in the presence of reaction delays and using only on-board sensors.

At a microscopic level, Chen *et al.* proposed a behavioral car-following model based on empirical data that captures phantom jams (Chen, Laval, Zheng, & Ahn, 2012). Therein, the authors showed statistical correlation in drivers' behavior before and during traffic oscillations. However, no suggestions to avoid phantom jams were offered. To that end, Nishi *et al.* developed a framework for "jam-absorbing" driving in Nishi, Tomoeda, Shimura, and Nishinari (2013). A "jam-absorbing vehicle" appropriately varies its headway with the aim of mitigating phantom jams. This work was extended by Taniguchi, Nishi, Ezaki, and Nishinari (2015) to include car-following behavior. Therein, the authors also numerically constructed the region in parameter space that avoids formation of secondary jams.

In the context of platoon stability, it has been shown that well-placed, communicating autonomous vehicles may be used to stabilize platoons of human-driven vehicles (Orosz, 2016). More gen-