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Finite element computational procedure for convective flow of nanofluids in an annulus

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ABSTRACT

In the present study, the detailed procedures of Galerkin weighted residual technique of finite element method (FEM) for solving two-dimensional incompressible natural convective flow of nanofluids using nonhomogeneous dynamic model are discussed for the first time. The physical domain is discretized by using unstructured triangular elements. The governing partial differential equations of nanofluids are made dimensionless using the suitable transformation of variables for weak formulations. The method of weighted residuals is used for obtaining the approximate solutions. This approach typically leads to a sparse and indefinite matrix that is difficult to solve efficiently. The formation of an indefinite matrix is avoided in the present work by introducing an artificial compressibility term in the continuity equation. Unequal order interpolation functions are used for pressure, velocity, temperature and concentration variables. The coefficient matrices are calculated using interpolation functions. Assembling of triangular elements in the discretized domain is discussed elaborately. The process of calculating boundary integrals is also discussed. The Newton-Raphson iteration technique along with Euler-backward scheme is used to solve the global matrix. The sample results are obtained for the convective flow of nanofluids in a concentric annulus. It shows that the annulus of having higher thickness is the best performer enhancing convective heat transfer rates.

1. Introduction

Finite element method is the most powerful numerical method ever devised for the analysis of engineering problems for finding the approximate solutions of a system of partial differential equations. The popularity of this method enhances over the course of time for solving fluid dynamics problems. To deal a range of unsteady and nonlinear flow problems in irregular domains, this method is adequately universal. The fundamental characteristic of finite element approximations is the generation of a mathematical model by patching up together a number of purely local approximations of the phenomena under consideration. This feature of the method successfully emancipates the experts from conventional troubles related to asymmetrical geometries, mixed boundary conditions, and multi-associated domains. Furthermore, applications have been firmly rooted in the physics of the problem at hand. The ability to deal with the arbitrary geometries is an important advantage of finite element methods. Also, the grids can easily redefine and each element can simply subdivide. Mathematically, finite element method is comparatively easy to analyze and can be shown to have optimality properties for certain types of equations. The preliminary studies indicate that the resulting equations are better conditioned than those obtained by finite difference approximations of the governing equations for a given order of accuracy (Oden and Wellford [1]).

In FEM, the domain is divided into a set of discrete volumes of finite elements that are generally unstructured. In 2D, the finite elements are usually constructed by triangles or quadrilaterals whereas, in 3D, tetrahedral or hexahedra are most often used. The distinctive feature of weighted residual finite element method is that the equations are multiplied by a weight function before they are integrated over the entire domain. The solution is approximated by a linear shape function within each element in a way that guarantees continuity of a solution across element boundaries. Such a function can be constructed from its values at the corners of the elements. The weight function is usually of the same form of shape function. Then the approximation is substituted into the weighted integral of the conservation equations. The equations to be solved are derived by requiring the derivative of the integral with respect to each nodal value to be zero; this corresponds to selecting the best solution within the set of allowed functions. The result is a set of non-linear algebraic equations.

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Nomenclature

A	area of an element
B	applied magnetic field ($Nm^{-1} A^{-1}$)
C	concentration of nanofluid ($mol\ m^{-3}$)
C_C	reference concentration ($mol\ m^3$)
C_i	constant terms (i = 1–8)
c_p	specific heat ($J\ kg^{-1} K^{-1}$)
D_B	Brownian diffusion coefficient ($m^2\ s^{-1}$)
d_p	diameter of nanoparticle (nm)
D_T	thermal diffusion coefficient ($m^2\ s^{-1}$)
D_T^{ℓ}	numerical value of D_T
g	acceleration due to gravity ($m\ s^{-2}$)
h	heat transfer coefficient of nanofluid ($Wm^{-2} K^{-1}$)
H	weight function
$[H]$	matrix of linear coordinates of an element
$\mathbf{J}$	Jacobian matrix or tangent stiffness
k_B	Boltzmann constant (JK^{-1})
Le	Lewis number
L_i	area coordinates
$\mathbf{n}$	unit normal vector
N_{TBTC}	dynamic diffusion parameter
N_{TBT}	dynamic thermo-diffusion parameter
Nu	Nusselt number
n_x,n_y	direction cosines
N_β	element shape function
p	dimensional modified pressure (Pa)
P	dimensionless modified pressure
Pr	Prandtl number
r	thickness of annulus
R_1	radius of inner circle
R_2	radius of outer circle
Ra_T	Rayleigh number
Ra_C	modified Rayleigh number
Sc	Schmidt number
Ha	Hartmann number
H_λ	element shape function for pressure
t	dimensional time (s)
T	nanofluid temperature (K)
T_C	reference temperature (K)
U,V	dimensionless nanofluid velocity
(u,v)	dimensional nanofluid velocity (ms^{-1})
V_T	thermophoretic velocity (ms^{-1})
W	weight function
X,Y	dimensionless coordinates

Greek symbols

α	thermal diffusivity ($m^2 s^{-1}$)
β	thermal expansion coefficient (K^{-1})
β^*	mass expansion coefficient (mol^{-1})
ϕ	nanoparticles volume fraction
Φ	dimensionless concentration
Ψ	sphericity of the nanoparticle
μ	viscosity ($kg\ m^{-1}\ s^{-1}$)
ν	kinematic viscosity($m^2 s^{-1}$)
κ	thermal conductivity ($Wm^{-1} K^{-1}$)
σ	electrical conductivity (Sm^{-1})
ΔC	film concentration drop ($mol\ m^{-3}$)
ΔT	film temperature drop (K)
Δt	nominal time difference
θ	dimensionless temperature
θ_c	temperature at cold wall
θ_h	temperature at hot wall
ρ	density ($kg\ m^{-3}$)
ε	convergence criterion
λ^*	correction factor
ξ	dimensionless time

Subscripts

bf	base fluid
p	nanoparticles
nf	nanofluid
ave	average
t	derivative with respect to time
x	derivative with respect to x
y	derivative with respect to y
xx	double derivative with respect to x
yy	double derivative with respect to y

The central ideas of the finite element method can be traced back to the work of Hrennikoff [2] and Courant [3]. However, the formal presentation of the method is generally attributed by the paper of Turner et al. [4]. While the method has found wide application in solid and structural mechanics, its application to flow problems has come only in rather recent times. Primary practices of the method were continuously accompanying with variational statements of the problem under attention so that it is natural that steady, potential flow problems were the first solved using finite element method. In this respect, the works of Zienkiewicz et al. [5] and Martin [6] are worth noticing. Finite element models of unsteady compressible and incompressible flow problems were obtained by Oden and Somogyi [7] and Oden [8]. The implementations of finite element method to a number of significant problems in fluid mechanics have been described by Argyris [9], Tong [10], Reddi [11], Baker [12]. The book of Zienkiewicz [13], Strang and Fix [14], Codina [15], Zienkiewicz et al. [16] and Reddy [17] can be consulted for additional references.

There are many benefits of the Galerkin weighted finite element method. The main advantage of this method for the eigenvalue problems is its capability to deliver tremendously controlling numerical tackles to resolve problems with additional particularized geometries, physical disseminations, and so forth that barely can be dealt by other approaches. The Galerkin weighted method is used as a global method in the mechanical systems to reduce the complexity of the set of partial differential equations to ordinary differential equations. Usually the variational element shape function is used as basis function which means that any polynomial or the suitable element shape function can be used if the mode shapes are difficult to bring. One way, this method reduces the dimensionality of the problem hence it is much faster.

Many scholarly articles are published using weighted residual finite element method based on different packages or software. Nevertheless, the step by step weighted residual finite element procedure by the interpolation shape function along with the solutions steps over a coupled set of five partial differential equations for continuity, momentum, energy, and concentration is very rare in the heat transfer analysis. Thus, the motivation of our work to present the procedures of Galerkin weighted residual technique of finite element method to solve a two-dimensional problem of natural convection incompressible flow of nanofluids in an annulus. The major steps involved in weighted residual finite element method of a typical problem are:

1. Discretization of the domain into a set of finite elements (mesh generation)
2. Weighted-integral or weak formulation of the partial differential equations to be analyzed.
3. Development of the finite element model of the problem using its